

**Unraveling trend and stationary components of
total factor productivity**

Franz C. Palm and Gerard A. Pfann 93-002

MERIT

Maastricht Economic Research Institute on Innovation and Technology

UNRAVELING TREND AND STATIONARY COMPONENTS OF TOTAL FACTOR PRODUCTIVITY

by

Franz C. Palm and Gerard A. Pfann *

January 1993

ABSTRACT

We propose a methodology to analyze the dynamic features of total factor productivity (TFP). Factor efficiency is assumed to evolve according to an unobserved component model which has the form of a dynamic version of factor analysis and which nests most of the specifications for technology shocks used in the literature. We show under which conditions the processes generating the components can be identified from measurements of TFP only. We also discuss how additional information on the technology shocks can be obtained from the factor demand equations associated with the production technology.

The methodology is applied to quarterly data for the manufacturing industry in the Netherlands, 1971.I-1990.IV. Technology is found to be driven by a stochastic trend with a first order moving average part. This finding is similar to results obtained by Lippi and Reichlin (1990) among others. The findings for TFP corroborate earlier findings for the dynamic features of technology in labor and capital demand equations for the manufacturing industry in the Netherlands.

JEL-code : 110, 210.

Keywords : Solow residual, unobserved components, identification.

Correspondence address : University of Limburg, Faculty of Economics and Business Administration, P.O. Box 616, 6200 MD Maastricht, The Netherlands.

We thank Dominique Guellec and Thijs ten Raaij for helpful comments. Financial support by the Royal Netherlands Academy of Arts and Sciences is gratefully acknowledged.

1. INTRODUCTION

Most research on the productivity growth is done along the lines of Solow (1957). To measure the effect of Hicks neutral technological change, one can subtract a Divisia index of input growths from output growths to obtain the Solow residual, an index of total factor productivity (TFP), that measures the accumulated effect of technology shocks. In real business cycle models, the variability of the Solow-residual is considered as a good proxy for the variability of exogenous technology shocks that drives the economy's cyclical behavior (Kydland and Prescott, 1982). Moreover, if growth is assumed to originate with technology shocks, all trending variables in the economy should share the same trend representing the accumulated nonstationary TFP. For example, in earlier work on manufacturing factor demand in the Netherlands, the U.K. and the U.S. we found that factor input, relative factor prices and TFP are cointegrated with labor and capital respectively.¹⁾

Alternatively, technology shocks can be interpreted as Marshallian productive externalities that give rise to increasing returns to scale (see e.g. Caballero and Lyons, 1990). In theory, plausible modifications of technology shock-driven as well as increasing returns economies can produce similar characteristics of macroeconomic variables (Murphy, Shleifer and Vishny, 1989). Blanchard and Quah (1989) identify permanent and transitory shocks from the joint dynamic processes of output and unemployment as technology and demand shocks respectively. Much of the recent debate in the econometric literature has been concerned with questions whether technological change is nonstationary and if it is, whether it should be modeled as a deterministic or a stochastic trend and whether one should allow for serial correlation in addition to a unit root. In the literature on industrial organization, to address questions about the internal or external nature of returns to scale, the impact of technology shocks on production has to be appropriately measured and modeled.

In this paper, the starting point is a continuous production function with two factors of production, labor and capital, and a stochastic component. This simple specification is chosen to illustrate how and when we can identify the parameters of the stationary and nonstationary processes that characterize factor efficiency. TFP is assumed to consist of an economy wide nonstationary part, and idiosyncratic stationary components of capital and labor efficiencies. The methodology developed in this paper can be applied to all types of continuous production functions for which a consistent estimate of TFP can be obtained. Here it is applied to quarterly data for the Netherlands manufacturing sector, 1971.I - 1990.IV.

The plan of the paper is as follows. In sections 2 and 3, the theoretical framework is presented, the identification of the time-series properties of the nonstationary component of TFP is examined and illustrated with an empirical example. In section 4 and 5 we explain that additional information, that can be obtained from first order necessary conditions for intertemporal profit maximization, is necessary to identify the bivariate stationary process of the serially correlated labor and capital efficiencies. We find that the volatility of the stationary components of TFP exceeds that of the trend component by an average factor

of ten. The drift parameter plays an important role in the factor demand equations. The non-stationarity of labor and capital is, just like TFP, a combination of a unit root and a trend component. Moreover, the variance of stationary shocks in employment exceeds that of capital, which is conform the stylized fact that capital is a more "fixed" (in the terms of Oi, 1962) factor of production than employment.

2. MODELING TOTAL FACTOR PRODUCTIVITY

To establish a basis for assessing the impact of technological change on production possibilities, one needs an analytically meaningful way of accounting for the growth at the level of the production unit. Consider a firm operating at any moment of time using a continuous production function which reads as follows

$$Y_t = F(L_t, K_t, E_{1t}, E_{2t}), \quad (2.1)$$

where Y_t , L_t , and K_t denote respectively production, labor and capital and E_{1t} and E_{2t} represent the efficiencies of labor and capital respectively. Furthermore, we assume F is homogeneous of degree γ_t in L_t and K_t . The total differential of $y = \ln Y$ can be written as

$$dy_t = \alpha_t d l_t + \beta_t d k_t + \delta_{1t} d z_{1t} + \delta_{2t} d z_{2t}, \quad (2.2)$$

where we use the notation $\alpha_t = F_L L/Y$ and $\beta_t = F_K K/Y$, $\delta_{1t} = F_{E_1} E_1/Y$ and $F_{E_2} E_2/Y$ with $F_x = \partial F / \partial X$, $l_t = \ln L_t$, $k_t = \ln K_t$, $z_{it} = \ln E_{it}$. When the arguments of the production function (2.1) are $L_t E_{1t}$ and $K_t E_{2t}$, the restrictions $\alpha_t = \delta_{1t}$ and $\beta_t = \delta_{2t}$ hold.

The random shocks z_{it} are assumed to have the following structure

$$z_{it} = \mu_t + (1 - \rho_i L)^{-1} \epsilon_{it}, \quad \epsilon_t = (\epsilon_{1t}, \epsilon_{2t})' \sim N(0, \Sigma) \quad (2.3a)$$

$$\text{with } \mu_t = \tau + \mu_{t-1} + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2), \quad (2.3b)$$

where μ_t is generated by a random walk with variance σ_η^2 and drift parameter τ and ϵ_{it} denotes the idiosyncratic component of factor i , $|\rho_i| < 1$ and L is the lag operator. The components η_t and ϵ_t are serially uncorrelated and uncorrelated with each other. We shall allow ϵ_{1t} and ϵ_{2t} to be correlated.

The unobserved component specification (2.3) has been chosen such that many of the processes used in the literature to model total factor productivity (TFP) are nested in it. For instance when $\sigma_1^2 = \sigma_2^2 = \sigma_\eta^2 = 0$, TFP can be represented by a linear deterministic trend with slope coefficient τ . For instance, David and Van de Klundert (1965) included a deterministic trend in a CES production function in order to measure the impact of technological change on factor efficiencies. If $\tau = 0$ and $\sigma_1^2 = \sigma_2^2 = 0$, TFP is generated by a random walk without drift, a model which is in line with that put forward by e.g. Lippi and Reichlin (1990). A stationary AR(1) process with ρ less than but near one has been used by Kydland and Prescott (1982) to model the persistence of the permanent component of the technology shock while the transitory component is white noise. Notice that the variables in their model have been detrended using

the Hodrick-Prescott filter so that there is no need to include a nonstationary component in their specification. The specification (2.3) accounts for a deterministic and/or a stochastic trend as well for a possible transitory component. For instance if one of the ρ_i 's is zero, the associated variable ϵ_{it} is white noise and is observationally equivalent to a transitory component. Moreover, one can interpret the nonstationarity part as reflecting the impact of radical innovations on TFP (see e.g. Aghion and Howitt (1992)) and the stationary part as representing the effects of incremental innovations. This interpretation is related to the notion that growth would die out if there were no radical innovations, a fact recognized in the literature (see e.g. Freeman (1989)). Assume in first instance that the coefficients α_t and β_t are known and constant, that $\alpha = \delta_1$ and $\beta = \delta_2$ and that differentials can be replaced by first differences. Then the first differences, Δ , of the so-called Solow residual u_t can be expressed as

$$\Delta u_t = \alpha \Delta z_{1t} + \beta \Delta z_{2t}. \quad (2.4)$$

An unobserved components structure for the technology shocks has also been proposed by Caballero and Lyons (1990). Note, for instance, that if F is a Cobb-Douglas production function with constant factor elasticities α and β and with discrete time parameter t

$$Y_t = A_t L_t^\alpha K_t^\beta \quad (2.5)$$

$$A_t = A \exp(\alpha z_{1t} + \beta z_{2t}) = A \exp(u_t),$$

and $\Delta \log A_t$ is the Solow residual.

Alternatively, the production technology can exhibit increasing returns due to Marshallian productive externalities ϕ_t , considered by Murphy, Shleifer and Vishny (1989) and Caballero and Lyons (1990)

$$A_t = \phi_t \exp(\alpha z_{1t} + \beta z_{2t}) = \phi_t \exp(u_t)$$

$$\phi_t = (Y_t^A \exp(-\mu_t))^{\phi_1}, \quad \phi_1 > 0$$

where Y_t^A is the aggregate output (e.g. at the macroeconomic level), so if aggregate economic activity rises relative to the trend the economy as a whole, and consequently the manufacturing sector, become more productive. Here, the increasing returns are a determinant of the rate of technology change and lead to an additional time-dependent component of TFP.²⁾

Given A_t or TFP, a natural question to ask is whether the processes for the components of u_t can be identified from information on u_t only. Substituting first differences of (2.3a) and (2.3b) into (2.4) and premultiplying (2.4) by the lag polynomials in the denominator yields the following result

$$\begin{aligned} (1 - \rho_1 L)(1 - \rho_2 L) \Delta u_t &= (\alpha + \beta)(1 - \rho_1)(1 - \rho_2) \tau \\ &+ (1 - \rho_1 L)(1 - \rho_2 L) \eta_t + \alpha(1 - \rho_2 L) \Delta \epsilon_{1t} + \beta(1 - \rho_1 L) \Delta \epsilon_{2t}. \end{aligned} \quad (2.6)$$

From expression (2.6), it becomes obvious that the residual u_t is generated by an ARIMA(2,1,2) process. This implication which can be checked in a direct way against the information in data on u_t , follows from

the fact that the right-hand side of (2.6) is a sum of three moving averages of order two. Also, from (2.6), it follows that the coefficients ρ_1 and ρ_2 are identical to the roots of the autoregressive part. Therefore, ρ_1 and ρ_2 are identifiable from information in the series u_t . Given that the parameters ρ_i are identified, the drift parameter r can be obtained from the intercept in the ARIMA model (2.6). Finally, to identify the variances σ_η^2 , σ_1^2 and σ_2^2 , we can use the information on the variance and the first and second order covariances of the moving average (MA) part

$$w_t = (1 - \rho_1 L)(1 - \rho_2 L)\eta_t + \alpha(1 - \rho_2 L)\Delta\epsilon_{1t} + \beta(1 - \rho_1 L)\Delta\epsilon_{2t}, \quad (2.7a)$$

which can be reparameterized as

$$w_t = (1 - \theta_1 L - \theta_2 L^2)v_t, \quad (2.7b)$$

where v_t is a white noise with expectation zero and constant variance σ_v^2 . The relationships between the parameters of the MA part (2.7b) and the parameters of interest σ_η^2 , σ_1^2 and σ_2^2 are

$$\begin{aligned} E[w_t^2] &= (1 + \theta_1^2 + \theta_2^2)\sigma_v^2 \\ &= (1 + (\rho_1 + \rho_2)^2 + (\rho_1\rho_2)^2)\sigma_\eta^2 + \\ &\quad 2\alpha^2(1 + \rho_2^2)\sigma_1^2 + 2\beta^2(1 + \rho_1^2)\sigma_2^2 + \\ &\quad \alpha\beta(\rho_1 + \rho_2 + 2(1 + \rho_1\rho_2))\sigma_{12}. \end{aligned} \quad (2.8a)$$

$$\begin{aligned} E[w_t w_{t-1}] &= \theta_1(\theta_2 - 1)\sigma_v^2 \\ &= -(\rho_1 + \rho_2)(1 + \rho_1\rho_2)\sigma_\eta^2 \\ &\quad - \alpha^2(1 + \rho_2)^2\sigma_1^2 - \beta^2(1 + \rho_1)^2\sigma_2^2 \\ &\quad - 2\alpha\beta(1 + \rho_1 + \rho_2 + \rho_2 + \rho_1\rho_2)\sigma_{12}. \end{aligned} \quad (2.8b)$$

$$\begin{aligned} E[w_t w_{t-2}] &= -\theta_2\sigma_v^2 = \rho_1\rho_2\sigma_\eta^2 + \alpha^2\rho_2\sigma_1^2 + \beta^2\rho_1\sigma_2^2 \\ &\quad + \alpha\beta(\rho_1 + \rho_2)\sigma_{12}. \end{aligned} \quad (2.8c)$$

Notice that the set of equations (2.8) is linear in σ_η^2 , σ_1^2 , and σ_{12} . When $\sigma_{12} = 0$, given that α and β are known (or can be consistently estimated) and that ρ_1 and ρ_2 are identifiable, the system (2.8) can be solved for σ_η^2 , σ_1^2 and σ_2^2 , and we conclude that these parameters are identified. If we allowed for contemporaneous correlation between ϵ_{1t} and ϵ_{2t} as in (2.8), we have a system of three linear equations in four unknowns σ_η^2 , σ_1^2 , σ_2^2 and σ_{12} . In that case, the process for the component η_t is identified, but the process for $(\epsilon_{1t}, \epsilon_{2t})$ is not.

Similarly, if the idiosyncratic components are serially correlated, for instance, if they are generated by a first order vector autoregressive (AR) process, the Solow residual becomes

$$u_t = (\alpha + \beta)\mu_t + (\alpha \beta)(I - RL)^{-1} \epsilon_t, \quad (2.9)$$

where R is the matrix of the AR part. First differencing and premultiplying (2.9) by the determinant of $(I - RL)$ yields, after substituting expression (2.3b) for $\Delta\mu_t$,

$$\begin{aligned} & [(1 - \rho_1 L)(1 - \rho_2 L) - \rho_{12}\rho_{21}L^2]\Delta u_t \\ &= (\alpha + \beta)[(1 - \rho_1)(1 - \rho_2) - \rho_{12}\rho_{21}]\tau \\ &+ [\alpha(1 - \rho_2 L) + \beta\rho_{21}L]\Delta\epsilon_{1t} + [\alpha\rho_{12}L + \beta(1 - \rho_1 L)]\Delta\epsilon_{2t} \\ &+ (\alpha + \beta)[(1 - \rho_1 L)(1 - \rho_2 L) - \rho_{12}\rho_{21}L^2]\eta_t. \end{aligned} \quad (2.10)$$

Also in this case, the TFP can be represented by an ARIMA(2,1,2) process. In terms of the order of the process, this case is observationally equivalent to that discussed above with diagonal matrix R . As is obvious from the representation (2.10), the parameters of the matrix R cannot be identified from the AR part. In fact we cannot achieve identification of the processes of the components from information in the series u_t only. As the variances of the components have to be nonnegative, at most inequality restrictions could be derived for the parameters of interest. Finally, it can be noticed that the specification with more than two uncorrelated first order autoregressive idiosyncratic components is identified.

The implications of these results are far-reaching. First whenever we can get consistent estimates of the Solow residual, we can test for the order of integration using for instance the Dickey-Fuller methodology. Second, we can also empirically determine the order of the univariate process for u_t or Δu_t if u_t appears to be integrated of order one. The order can be determined by using the Box-Jenkins approach or by applying a formal consistent model selection criterion.

The outcome of this analysis will either confirm subjective views about the process generating u_t or give hints about how to revise those prior beliefs. Third, if the component processes are identifiable we can estimate their parameters from the series u_t . If the parameters of the processes for the components cannot be identified, one will have to bring in additional information to achieve identification. The first order conditions for profit maximization of the firm contain extra information on the technology shocks. This information could be used to achieve identification of the parameters of interest. Notice that the first order conditions for profit maximization could also be used to estimate the parameters of the production function (see e.g. David and Van de Klundert (1965) and Caballero and Lyons (1990)). As the first order conditions of a dynamic optimization problem may be complicated and contrary to a suggestion of Caballero and Lyons (1990) may not be well approximated by a sequence of frictionless static problems, the direct approach based on information about TFP is attractive whenever parameter identification is achieved.

3. EMPIRICAL ANALYSIS OF THE NONSTATIONARY COMPONENT OF TFP

In this section, we apply the methodology to quarterly aggregate data for the manufacturing sector in the Netherlands, 1971.I-1990.IV. A description of the data and their sources are given in an appendix. The data are given in the figures A1 to A6 in the appendix.

Given observations on the labor income share, on output and input growth, along with Solow (1957), we can obtain the TFP as the rate change of output minus the Divisia index of input growth

$$\frac{\Delta \hat{A}_t}{\hat{A}_t} = \frac{\Delta Y_t}{Y_t} - \text{LIS}_t \cdot \frac{\Delta(L_t * H_T)}{L_t * H_T} - (1 - \text{LIS}_t) \cdot \frac{\Delta(K_t * U_t)}{K_t * U_t}, \quad (3.1)$$

where LIS_t denotes the labor income share. In absence of reasonably accurate series of users' cost of capital, we estimate the capital income share by $1 - \text{LIS}_t$.

Figures 1 and 2 show the series of TFP measured by $\hat{A}_t$ and of the labor income share in the value of total output for the sampling period. The series shown in figure 1 has been computed using the relationship (3.1) and setting $A_0 = 1$.

[FIGURES 1 AND 2]

Notice that in terms of the Cobb-Douglas specification (2.5) $A_t = A \exp(u_t)$ with $u_t = \alpha z_{1t} + \beta z_{2t}$, so that $u_t = \ln A_t - \ln A$. While the labor income share exhibits a decreasing trend in the second part of the observation period, A_t is subject to a strong positive trend on which fluctuations are imposed. The trend is appropriately accounted for by taking first differences or relative changes of A_t . There is little evidence for the presence of a segmented trend in A_t for the period of time under consideration.

Next we analyze the time series properties of $\hat{A}_t$ and $\Delta \hat{A}_t / \hat{A}_t$ being estimated from (3.1). The autocorrelation (AC) and partial autocorrelation (PAC) functions of $\Delta \hat{A}_t$ and $\Delta \hat{A}_t / \hat{A}_t$ are given in table 1, together with their standard errors (S.E.).

[TABLE 1]

The numbers in table 1 suggest an I(1) or an IMA(1,1) process for $\hat{A}_t$ and $\Delta \hat{A}_t / \hat{A}_t$. Estimation results for these models and for the more general ARIMA(1,1,1) specification are given in table 2.

[TABLE 2]

The likelihood ratio statistics, LR, of 3.267 and 1.179 in table 2 compare the I(1) model with the IMA(1,1) model. In terms of the t-test or the LR-test for the MA parameter, the IMA(1,1) model is preferred to the pure I(1) model for A_t . Also in terms of fit, the former model is the preferred one. For the ARIMA(1,1,1) model, none of the parameters of the lagged variables is significant. The results in table 2 suggest that $\hat{A}_t$ could be modeled as an IMA(1,1) process. For $\Delta \hat{A}_t / \hat{A}_t$, the IMA(1,1) model is not preferred to the I(1) model on statistical grounds although there is

some first order serial in the series. In the ARIMA(1,1,1) model there appear to be two common factors which cancel against each other and yield a model which is close to the IMA(1,1) model. Note that the constant term $\hat{\tau}$ is always significantly different from zero, but so close to zero that it will hardly be picked up by statistical tests for unit roots. However, it is just $\hat{\tau}$ that is responsible for the deterministic trend in TFP.

[TABLE 3]

Table 3 reports the Box-Pierce statistic and its p-value for the residuals of both series. In the first column, the number of residual autocorrelations included in the test statistic is given. From these results, it can be concluded that the I(1) model exhibits some residual serial correlation for high order lags. For the IMA(1,1) and the ARIMA(1,1,1) models, no significant residual autocorrelation is found. From these findings, it can be concluded that the IMA(1,1) model is a parsimonious representation of the serial dependence in both series.

How should these findings be interpreted in the framework of section 2.1 ? Under the assumption that $\rho_1 = \rho_2 = 0$, according to model (2.6), Δu_t is generated by an IMA(1,1) process. The sample moments (2.8) of the MA part of Δu_t are

$$\begin{aligned} E[w_t^2] &= \sigma_\eta^2 + 2[\alpha^2\sigma_1^2 + 2\alpha\beta\sigma_{12} + \beta^2\sigma_2^2] \\ E[w_t w_{t-1}] &= -[\alpha^2\sigma_1^2 + 2\alpha\beta\sigma_{12} + \beta^2\sigma_2^2], \end{aligned} \quad (3.2)$$

where $w_t = \Delta u_t - \tau$. From (3.2), we see that in this case $\sigma_\eta^2 = E[w_t^2] + 2E[w_t w_{t-1}]$ is identified. Estimates of the variance and first autocovariance of w_t are

$$\begin{aligned} E[\hat{w}_t^2] &= .35997E-3 \\ E[\hat{w}_t \hat{w}_{t-1}] &= -.04223E-3, \end{aligned} \quad (3.3)$$

so that

$$\hat{\sigma}_\eta^2 = .27551E-3.$$

From this analysis, we conclude that for the manufacturing sector in the Netherlands, TFP has been nonstationary in the seventies and eighties. Its trend is found to be stochastic. Also, one should allow for idiosyncratic effects on the two production factor labor and capital. These idiosyncratic effects appear to be negatively correlated but serially uncorrelated. Besides the first order moving average, which results from differencing TFP to render it stationary, there is not much need for autocorrelation in the technology shocks in Dutch manufacturing. This finding is at variance with the assumption made in some real business cycle models where technology shocks follow a first order autoregressive process. For instance, on detrended U.S. data, Kydland and Prescott (1982) use a first order autoregression with ρ close to one to represent the time dependence of technology shocks. The autoregression may well be an

artefact resulting from detrending using the Hodrick-Prescott filter (see Cogley and Nason (1992)).

4. ANALYSIS OF THE STATIONARY COMPONENT OF TFP

To identify the parameter of the stationary component of TFP we use the linear recursive model of the production technology that is characterized as interrelated factor demand under uncertainty with costs of adjustment (see Hansen and Sargent, 1990). We assume that the real factor prices are set before the factor input is decided on, an assumption which has been found appropriate in previous work for data for the Netherlands (see e.g. Palm and Pfann (1991)).

The empirical generating process of the real factor price (2x1)-vector $p_t = (W_t, Q_t)'$ deflated by the output price is found to be ARI(1,1)

$$(1-ML)\Delta p_t = c^p + \epsilon_t^p \quad \epsilon_t^p \sim ID(0, \Sigma_p) \quad (4.1)$$

with $c^p = (c^w, c^q)'$ being a (2x1)-vector and M a (2x2)-matrix of constant parameters. Given (4.1), the dynamic process for interrelated factor demand with adjustment costs can be written as a vector ARIMA model of the decision set $x_t = (L_t, K_t)'$ with the change in real factor prices as explanatory variables. This model yields

$$(I-RL)(I-AL)\Delta x_t = (I-RL)\Gamma(I-M)^{-1}\Delta p_t + c^x + \nu_t^x \quad (4.2)$$

$$\nu_t^x = (I-RL)\Gamma(\iota\eta_t + \Delta\epsilon_t), \quad (4.3)$$

where the (2x2)-matrix R is a linear transformation of the autoregressive matrix of technology shocks z_t ; the (2x2)-matrix A is the Kollintzas (1985) interrelation matrix; Γ is a (2x2)-matrix with constant parameters, ι is the (2x1) unit root vector and c^x is the (2x1) intercept vector, that includes the technology drift parameter τ .³⁾

In the foregoing analysis, we found that the autoregressive (2x2)-matrix R of the stationary component of TFP is empirically close to zero. Therefore, in equations (4.2) and (4.3) R will be set to zero. Substituting (4.1) into (4.2) we obtain a system of linearized decision rules being the estimable vector ARIMAX(1,1,2) process for interrelated factor demand

$$(1-AL)\Delta x_t = \Gamma(I-M)^{-1}M\Delta p_{t-1} + c + \nu_t \quad (4.4)$$

$$\nu_t = \Gamma(\iota\eta_t + \Delta\epsilon_t + (I-M)^{-1}\epsilon_t^p) \quad (4.5)$$

where $c = \Gamma((I-M)^{-1}c^p + c^x)$.

We assume

$$\eta_t \perp \epsilon_t^p \text{ and } \epsilon_t \perp \epsilon_t^p, \quad (4.6)$$

so that

$$E[\nu_t \nu_t'] = \sigma_\eta^2 [(\Gamma_t)(\Gamma_t)'] + 2\Gamma\Sigma_\epsilon\Gamma' + (\Gamma(I-M)^{-1})\Sigma_p(\Gamma(I-M)^{-1})' \quad (4.7a)$$

and

$$E[\nu_t \nu_{t-1}'] = -\Gamma\Sigma_\epsilon\Gamma'. \quad (4.7b)$$

Hence Σ_ϵ can be identified from (4.7b) if Γ can be consistently estimated. Σ_p can be identified from (4.7a) using that $\hat{\sigma}_\eta^2 = .27551 \text{ E-}3$, or Σ_p can be estimated separately from the estimation of (4.3).

To estimate the ARIMAX-system (4.3), (4.4), (4.5) and (4.6) we applied a 2-step residual-adjusted GLS estimation procedure which implements the Gauss-Newton algorithm and therefore yields consistent and efficient parameter estimates after one iteration provided consistent initial estimates are used (see e.g. Palm and Zellner (1980)). We used instrumental variables to obtain consistent initial estimates of the parameters of (4.4) and (4.5).⁴⁾ We did not restrict the matrix Λ to be diagonal.⁵⁾ However, the matrix Γ , estimated from (4.4) turned out to be empirically diagonal. In addition to these restriction, the estimation of Σ_ϵ requires an additional assumption, implicitly induced by the overidentifying restrictions in the reduced form model (4.4). To compute Σ_ϵ we can use the identity that follows directly from (4.7) and the moving average $(1 - \Psi L)\tilde{\epsilon}_t$ that will be estimated. This identity reads

$$\Gamma\Sigma_\epsilon\Gamma' = \Psi\tilde{\Sigma}, \quad \tilde{\Sigma} = E\tilde{\epsilon}_t\tilde{\epsilon}_t'$$

so that

$$\Sigma_\epsilon = (\Gamma)^{-1}\Psi\tilde{\Sigma}(\Gamma')^{-1}. \quad (4.8)$$

Consequently, the right hand side of (4.8) should be symmetric which is only true if $\Psi\tilde{\Sigma}$ is positive definite. Then we can write $\Psi\tilde{\Sigma} = \Phi'\Phi$, such that (4.8) becomes

$$\Sigma_\epsilon = (\Phi(\Gamma')^{-1})'(\Phi(\Gamma')^{-1})$$

which is symmetric. The implicit restrictions on the elements of Ψ implied by the symmetry of Σ_ϵ have also been taken into account in the 2-step residual-adjusted GLS estimation procedure. Table 4 presents the results.

[TABLE 4]

Given the estimates for the matrices Γ , Ψ and $\tilde{\Sigma}$, we can now compute Σ_ϵ from (4.8). We find

$$\hat{\Sigma}_\epsilon = 1.0\text{E-}3 * \begin{bmatrix} 5.018 & 1.111 \\ 1.165 & 3.447 \end{bmatrix}.$$

The estimation of the structural parameters produced well conditioned matrix estimates for Λ , Γ and Σ_ϵ . The determinants of Λ and Γ are positive (which is required through the underlying structural model) and Σ_ϵ is close to being symmetric. However, Hosking's multivariate portmanteau test

implies that the (Tx4) matrix of residual errors is autocorrelated. This may be solved through adding seasonal dummies or by loosening the restrictions (4.6).

The results we obtained for the TFP process are interesting and in line with one's intuition. The volatility of the stationary component of technology shocks σ_i^2 is much higher than the variance of the drift component μ_t , σ_η^2 . The drift parameter plays an important role in the factor demand equations. The non-stationarity of labor and capital is, just like TFP, a combination of a unit root and a trend component. Moreover, the variance of stationary shocks in employment exceeds that of capital, which is conform the stylized fact that the production factor capital is more "fixed" (in terms of Oi (1962)) than employment.

5. CONCLUDING REMARKS

In this paper, we proposed a methodology to analyze the dynamic features of total factor productivity. More specifically we put forward an unobserved components time series model (which is in fact a dynamic version of factor analysis) which nests most of the specifications used in the literature to represent the nonstationarity and serial correlation of TFP. We also showed that in some instances, the processes for the unobserved components can be identified from information on TFP only. For more complicated processes, information from the solution of the first order conditions of the intertemporal decision problem will be required in order to achieve full identification. Relying on the Euler equations only and using proxy variables for the unobserved expectational variables in these equations will generally not be sufficient to identify the processes of the components of TFP. Nevertheless, the Euler equations yield additional information about the type of nonstationarity of TFP.

In the empirical part, we illustrated the methodology by analyzing the dynamic properties of TFP for the manufacturing industry in the Netherlands, 1971-1990. To obtain an estimate of TFP, we used the method originally put forward by Solow (1957) which relies on the labor income share of the value of output and on the assumption of constant returns to scale.

The findings of an IMA(1,1) process for the technology shocks were in line with those of several studies as far as the nonstationarity is concerned. They are also consistent with the findings for the technology shocks using the closed form solution of an intertemporal decision model. They are at variance with the assumption of a first order autoregression which underlies some of the real business cycle models. Our findings for TFP also corroborated earlier results for the technology shocks in labor and capital demand models for the manufacturing sector in the Netherlands.

The direct approach of analyzing TFP is simpler than studying the properties of technology shocks using the Euler conditions of the decision model. It is also quite general, in the sense that the functional form of the production function need not be completely specified. Of course, if it is known, it would be natural to use it to estimate TFP. The fact that the findings for TFP and for the system of dynamic factor demand equations are

consistent with each other is an indication that the assumption of constant returns to scale when measuring TFP is probably appropriate as a first approximation. While for reason of lack of data, we had to make the assumption of constant returns to scale, one could also obtain TFP under alternative assumptions regarding returns to scale and market structure. At present, we confine ourselves to referring to Nelson (1986), Hall (1988,1990), Caballero and Lyons (1990), who proposed ways of accounting for variable returns to scale and market power when estimating TFP.

We acknowledge the importance of applying the methodology we have proposed in this paper to measure TFP when returns to scale are variable. Our study, however, has made it clear that in order to do so extra identifiable sources of dynamics are required. For example, additional assumptions can be imposed on the structural model, or data on the user costs of capital (which are notoriously untrustworthy) may be used. Also, when panel data on production are available our methodology could be extended along lines of Neusser (1992) to allow for spillovers from one sector to another.

Finally, we like to stress that it would be preferable to measure technology as an input instead of estimating it by computing TFP.

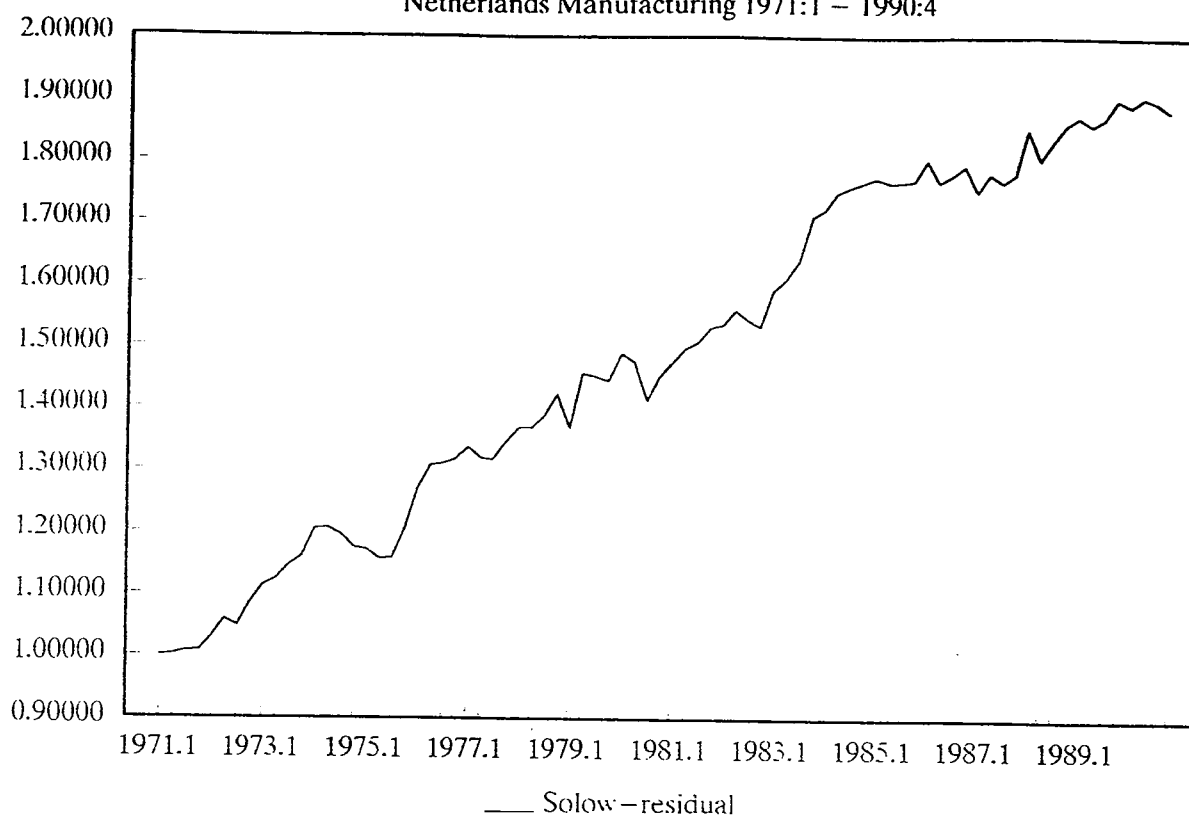
REFERENCES

- Aghion, P., and P. Howitt (1992), "A model of growth through creative destruction", Econometrica 60, 323-351.
- Blanchard, O.J., and D. Quah (1989), "The dynamics of aggregate demand and supply disturbances", American Economic Review 67, 101-115.
- Caballero, R.J., and R.K. Lyons (1990), "Internal versus external economies in European industry", European Economic Review 34, 805-826.
- Cogley, T., and J.M. Nason (1992), "Effects of the Hodrick-Prescott filter on trend and difference stationary time series : Implications for business cycle research", mimeo, University of California, Berkeley.
- David, P.A., and Th. van de Klundert (1965), "Biased efficiency growth and capital-labor substitution in the U.S., 1899-1960", American Economic Review 55, 357-394.
- Engle, R.F., and C.W.J. Granger (1987), "Co-integration and error-correction : Representation, estimation and testing", Econometrica 55, 251-276.
- Freeman, C. (1989), "The nature of innovation and the evolution of the productive system", paper prepared for the OECD International Seminar on Science, Technology and Economic Growth, 5-8 June 1989.
- Hall, R.E. (1988), "The relation between price and marginal cost in U.S. industry", Journal of Political Economy 96, 921-947.
- Hall, R.E. (1990), "Econometric research on shifts of production functions at medium and high frequencies", invited paper presented at the 1990 World Congress of the Econometric Society.
- Hansen, L.P., and T.J. Sargent (1990), "Recursive linear models of dynamic economies", manuscript.
- Kodde, D.A., Palm, F.C., and G.A. Pfann (1990), "Asymptotic least-squares estimation - Efficiency considerations and applications", Journal of Applied Econometrics 5, 229-243.
- Kollintzas, T. (1985), "The symmetric linear rational expectation model", Econometrica 53, 963-976.
- Kydland, F.E., and E.C. Prescott (1982), "Time to build and aggregate fluctuations", Econometrica 50, 1345-1370.
- Lippi, M., and L. Reichlin (1990), "Diffusion of technical change and the identification of the trend component in real GNP", OFCE, Document de travail 90-04, Paris.
- McCallum, B.T. (1976), "Rational expectations and the natural rate hypothesis : Some consistent estimates", Econometrica 44, 43-52.
- Murphy, K.M., Shleifer, A., and R.W. Vishny (1989), "Building blocks of market clearing business cycle models", in Blanchard, O.J., and S. Fischer (eds.), NBER Macroeconomics Annual 1989, Cambridge, The MIT Press, 247-287.
- Nelson, R.N. (1986), "Industry growth accounts and production functions when techniques are proprietary", paper presented at a MERIT-Conference.
- Neusser, K. (1992), "Dynamics of total factor productivities", University of Vienna, Working paper.
- Oi, W. (1962), "Labor as a quasi-fixed factor", Journal of Political Economy 70, 538-555.
- Palm, F., and A. Zellner (1980), "Large sample estimation and testing procedures for dynamic equation systems", Journal of Econometrics 12, 251-283.

- Palm, F.C., and G.A. Pfann (1991), "Interrelation, structural changes and cointegration in a model for manufacturing demand in the Netherlands", Recherches Economiques de Louvain 57, 221-243.
- Palm, F.C., Peeters, H.M.M., and G.A. Pfann (1992), "Adjustment costs and time-to-build in factor demand in the U.S. manufacturing industry", University of Limburg, Research memorandum 92-012.
- Pfann, G.A., and F.C. Palm (1992), "Asymmetric adjustment costs in non-linear labour demand models for the Netherlands and U.K. manufacturing sectors", forthcoming in Review of Economic Studies.
- Solow, R.M. (1957), "Technical change and the aggregate production function", Review of Economics and Statistics 39, 312-320.

Fig.1: Solow Residual

Netherlands Manufacturing 1971:1 – 1990:4

**Fig.2: Labor Income Share**

Netherlands Manufacturing 1971:1 – 1990:4

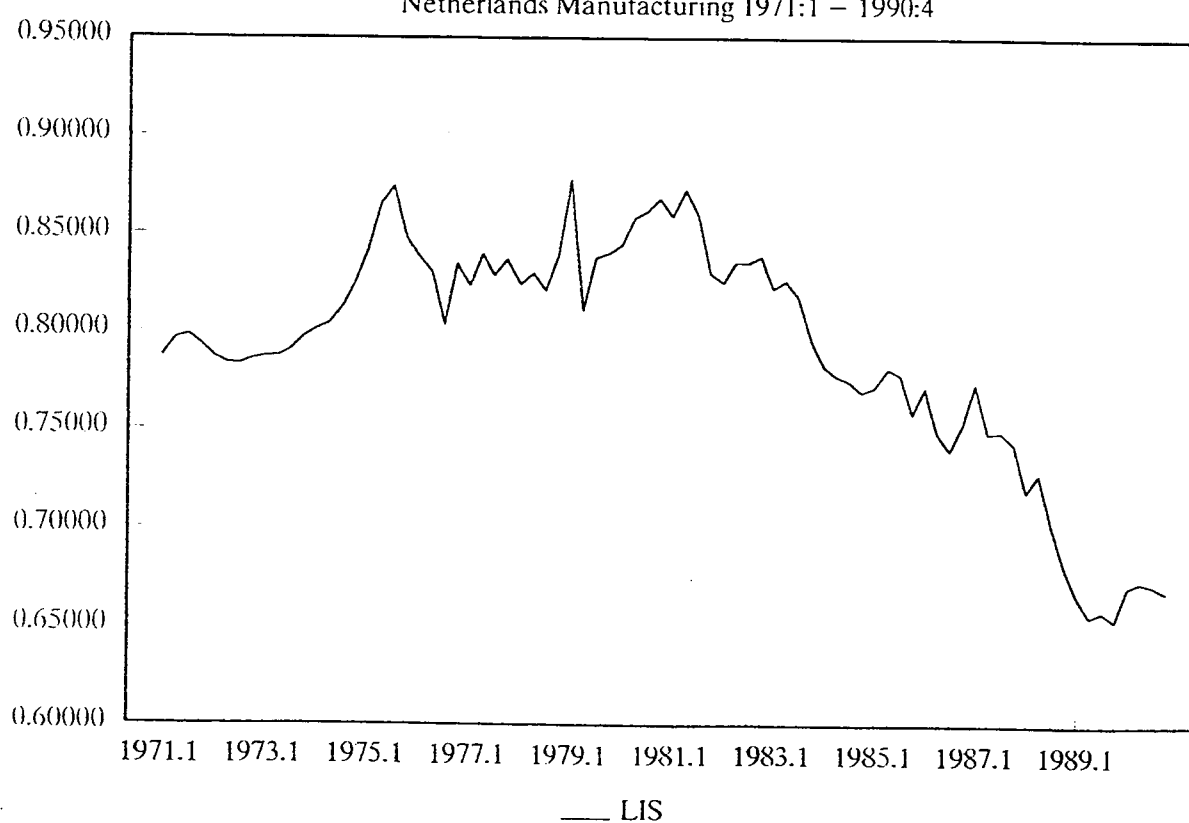


Table 1 : Autocorrelations and Partial-Autocorrelations of $\Delta\hat{A}_t$ and $(\Delta\hat{A}_t/A_t)$.[illegible]

Table 2 : Univariate Models for $\hat{A}_t$ and $\Delta\hat{A}_t/A_t$. *)

1. I(1)	$\Delta\hat{A}_t = .01117 + e_t$ (3.360) S.E. = .027350 RSS = .058346	$\hat{\tau}_1 = .01117$
2. IMA(1,1)	$\Delta\hat{A}_t = .011278 + (1-.206L)e_t$ (4.659) (1.827) S.E. = .026964 RSS = .055982	$\hat{\tau}_2 = .011278$ LR(1) = 3.267 p-value = .071
3. ARIMA(1,1,1)	$(1+.023L)\Delta\hat{A}_t = .011532 + (1-.184L)e_t$ (-.041) (1.709) (.334) S.E. = .027140 RSS = .055980	$\hat{\tau}_3 = .011532$ LR(1) = .0028 p-value = .958
4. I(1)	$\Delta\hat{A}_t/A_t = .00212 + e_t$ (3.769) S.E. = .018863 RSS = .027753	$\hat{\tau}_1 = .00212$
5. IMA(1,1)	$\Delta\hat{A}_t/A_t = .00804 + (1-.124L)e_t$ (4.321) (1.090) S.E. = .018844 RSS = .027342	$\hat{\tau}_2 = .00804$ LR(1) = 1.179 p-value = .277
6. ARIMA(1,1,1)	$(1-.747L)\Delta\hat{A}_t/A_t = .00206 + (1-.851L)e_t$ (2.738) (.945) (3.880) S.E. = .018894 RSS = .027132	$\hat{\tau}_3 = .00206$ LR(1) = .609 p-value = .435

*) t-values are given between parentheses.

Table 3 : Box-Pierce test.

Variable ΔA_t Lag	I(1)		IMA(1,1)		ARIMA(1,1,1)	
	BP	p-value	BP	p-value	BP	p-value
1	3.024	.082	.000	1.000	.000	1.000
2	3.258	.196	.028	.986	.037	.982
3	8.177	.042	3.011	.389	3.010	.390
4	13.676	.008	8.497	.075	8.482	.075
8	15.574	.049	12.504	.130	12.524	.129
12	24.614	.016	18.944	.090	18.981	.089
16	34.942	.004	26.191	.051	26.202	.051
20	41.251	.003	31.541	.048	31.571	.048
Variable $\Delta A_t/A_t$ Lag	I(1)		IMA(1,1)		ARIMA(1,1,1)	
	BP	p-value	BP	p-value	BP	p-value
1	1.125	.289	.000	1.000	.082	.774
2	1.289	.525	.047	.976	.101	.951
3	4.948	.176	2.692	.441	4.519	.211
4	9.361	.052	7.212	.125	7.487	.112
8	12.379	.135	11.605	.169	9.758	.282
12	20.527	.058	18.428	.103	17.186	.143
16	27.409	.037	23.685	.097	22.847	.118
20	32.788	.035	28.785	.092	27.579	.119

Table 4 : Efficient Residual-Adjusted GLS Estimation Results of ARIMAX System *

	Employment ΔL_t	Capital ΔK_t	Wage Cost ΔW_t	Investment Costs ΔQ_t
CONST	-.993E-2 [.000]	.659E-2 [.011]	.551E-2 [.006]	-.683E-3 [.664]
$\Lambda = (\lambda_{ij})$ $i, j = 1, 2$	$\lambda_{11} = .122$ [.199]	$\lambda_{21} = .034$ [.851]		
	$\lambda_{12} = .436$ [.000]	$\lambda_{22} = .393$ [.007]		
$\Gamma = (\gamma_{ii})$ $i = 1, 2$	$\gamma_{11} = .072$ [.028]	$\gamma_{22} = .079$ [.191]		
$M = (m_{ij})$ $i, j = 1, 2$			$m_{11} = .109$ [.477]	$m_{21} = .157$ [.192]
			$m_{12} = .557$ [.003]	$m_{22} = .397$ [.008]
MA-coef. $\Psi = (\psi_{ij})$ $i, j = 1, 2$	$\psi_{11} = .215$ [.002]	$\psi_{21} = .051$ [.000]		
	$\psi_{12} = .032$ [.000]	$\psi_{22} = .120$ [.368]		
$\tilde{\Sigma} = 1.0E-4 * \begin{bmatrix} 1.211 & \\ .035 & 1.771 \end{bmatrix} \quad \Sigma_p = 1.0E-3 * \begin{bmatrix} .227 & \\ .120 & .141 \end{bmatrix}$				
Hosking's Multivariate Portmanteau Test on Residual Autocorrelation [p-values]				
AR(5) = .175 AR(10) = .005 AR(15) = .001 AR(20) = .000				

* p-values are given in squared brackets.

APPENDIX : DESCRIPTION AND SOURCES OF QUARTERLY MANUFACTURING DATA IN THE NETHERLANDS (SBI 2/3)

The base year of all prices and indices is 1980. All data in the model have been standardized = dividing the series by the sample mean.

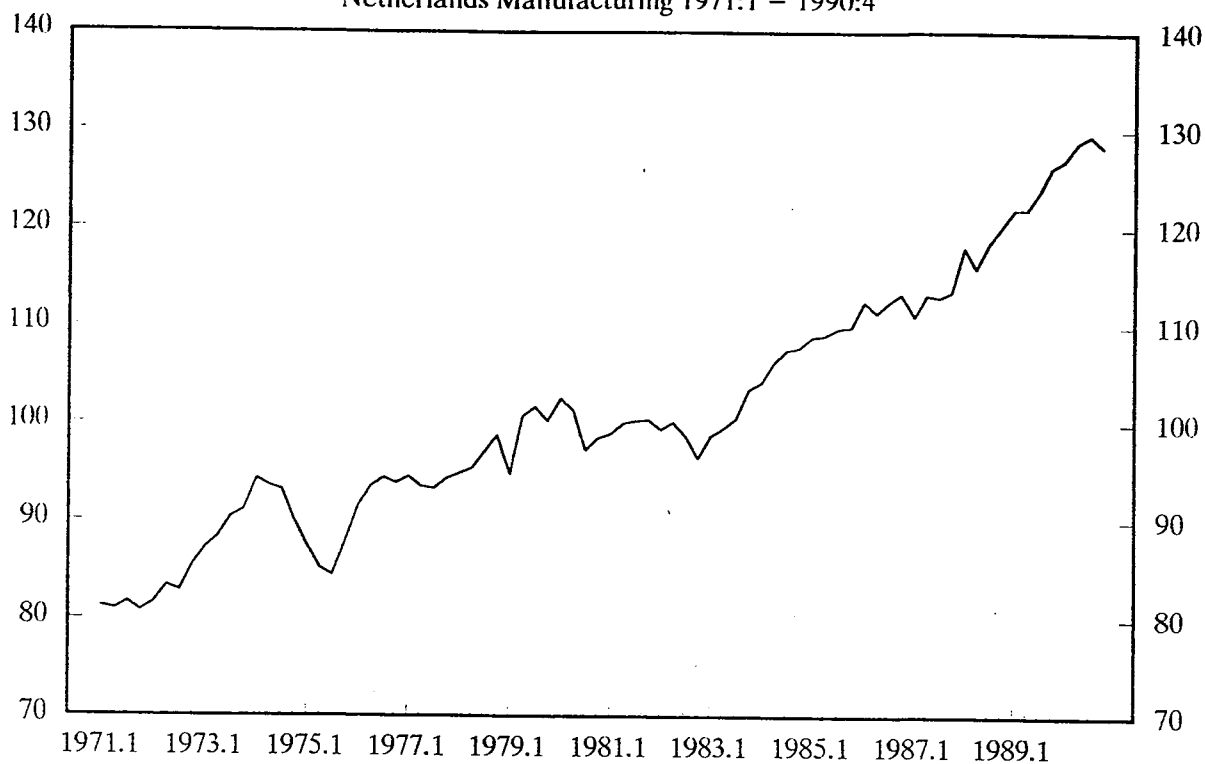
- Y_t is a seasonally adjusted quantity index of a quarterly average of daily industrial production (National Accounts, C.B.S., The Netherlands).
- P_y is the producer price index of domestically sold production (SBI 2/3).
- K is manufacturing sector (SBI 2/3) capital stock in constant 1980 prices. Annual data provided by the Central Planning Bureau; quarterly fluctuations resemble quarterly fluctuations of manufacturing gross investment in fixed assets. (Quarterly Accounts, C.B.S., The Netherlands).
- U is rate of capacity utilization (SBI 2/3).
- P_k is the producers price index of domestically placed investment goods. (Afzet Binnenland van de Industrie, SBI 2/3).
- Q is P_k/P_y .
- L is employment of 16-64 years old people working at least 20 hours per week (SBI 2/3).
- H is the average paid working time including overtime of all employees between 16 and 64 years old (SBI 2/3).
- P_l are the costs of labor in current prices per hour worked (SBI 2/3). Since the fourth quarter of 1986, this series is the quarterly updated wage cost. For the period previous to 1986.IV, this series is based on six month data on labor costs and quarterly percentage increases of wages per hour worked (SBI 2/3).
- W is P_l/P_y .
- LIS is the labor share of income (SBI 2/3) obtained from quarterly data in Algemene Industrie Statistiek en De Statistiek Werkzame Personen.

Equation (3.1) is computed as follows:

$$\frac{\Delta \hat{A}_t}{A_t} = \frac{\Delta Y_t}{Y_t} - LIS_t \cdot \frac{\Delta(L_t * H_t)}{L_t * H_t} - (1 - LIS_t) \cdot \frac{\Delta(K_t * U_t)}{K_t * U_t}.$$

Fig A1: Production Volume Index

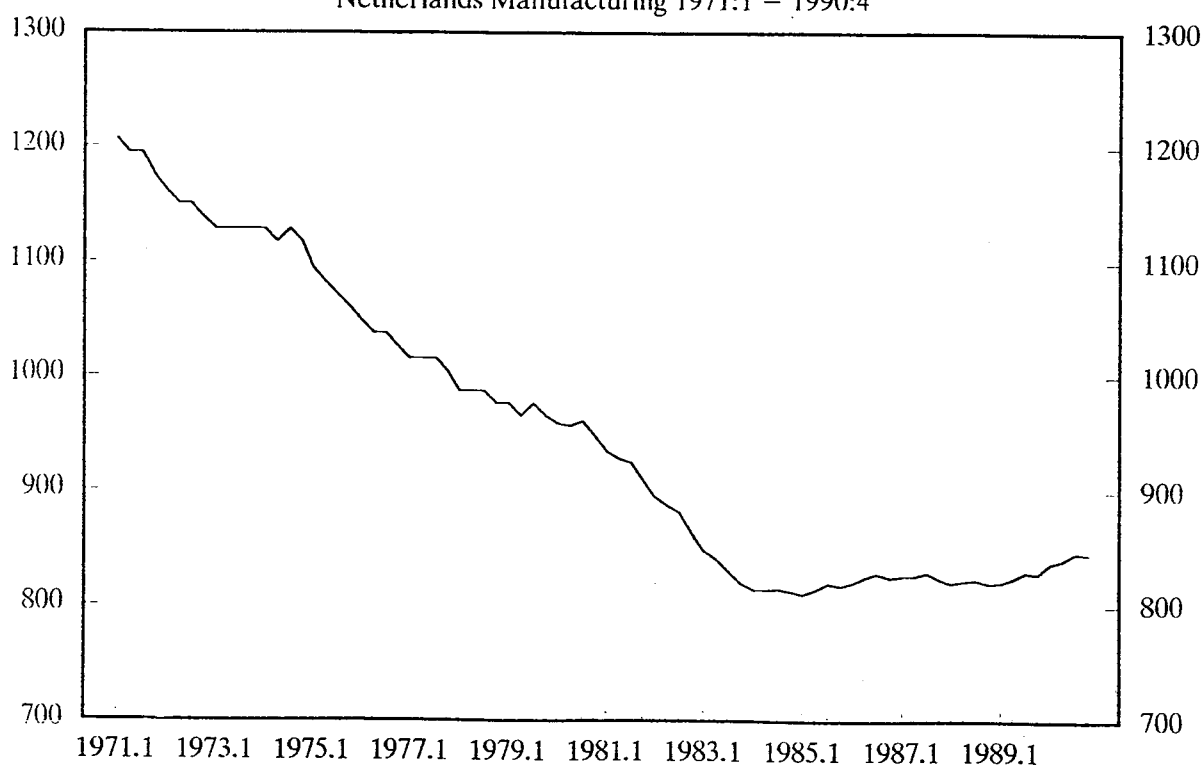
Netherlands Manufacturing 1971:1 – 1990:4



— Production Volume Index, Seasonally Adjusted (1980 = 100)

Fig A2: Employment

Netherlands Manufacturing 1971:1 – 1990:4



— 16 – 64 Years Old People Employed at Least 20 Hours per Week (x 1,000)

Fig A3: Hourly Wage Costs

Netherlands Manufacturing 1971:1 – 1990:4

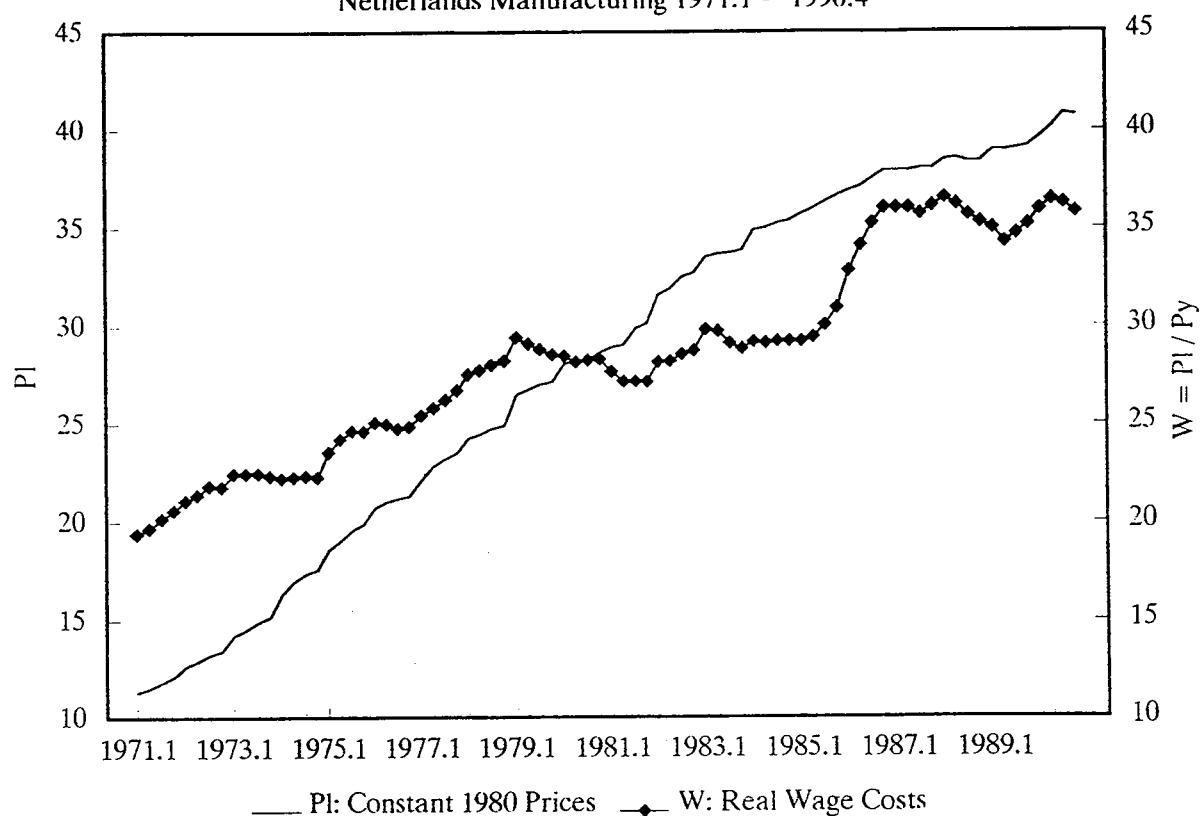


Fig A4: Capital Stock

Netherlands Manufacturing 1971:1 – 1990:4

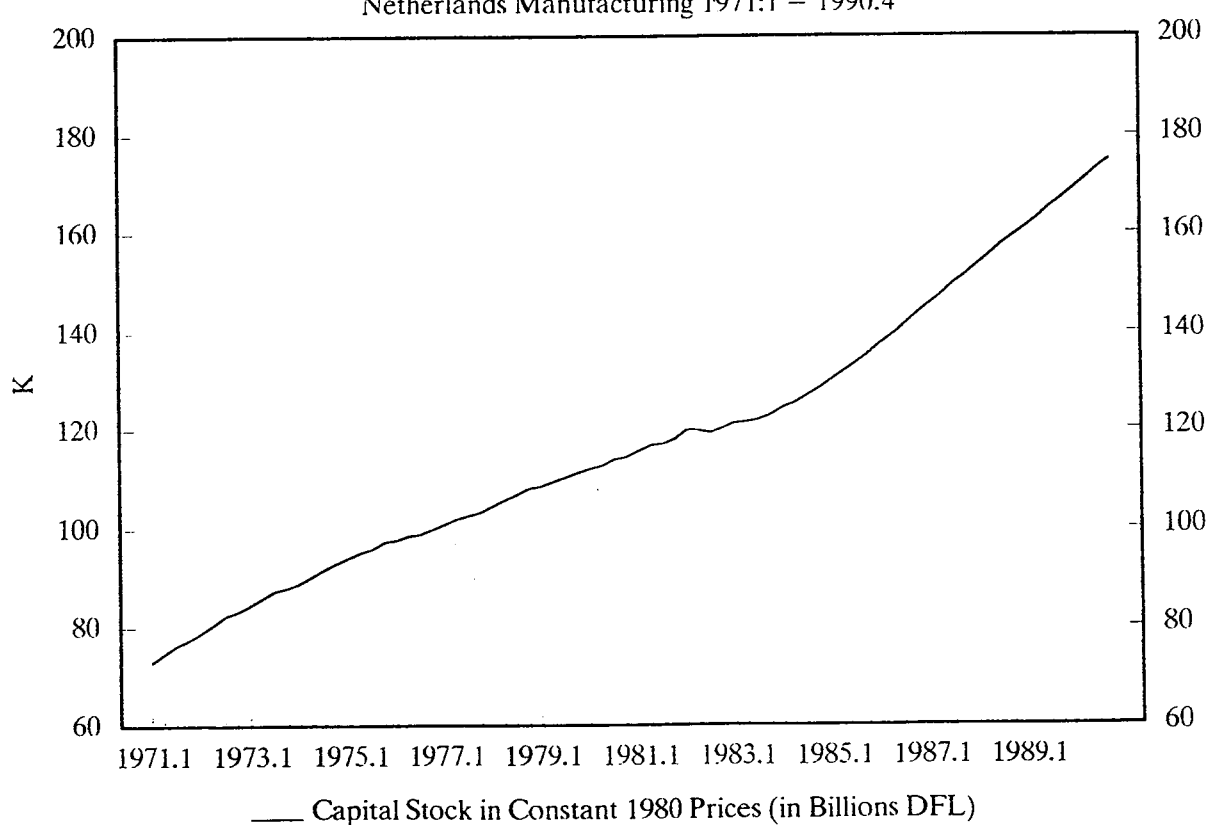


Fig A5: Investment Costs

Netherlands Manufacturing 1971:1 – 1990:4

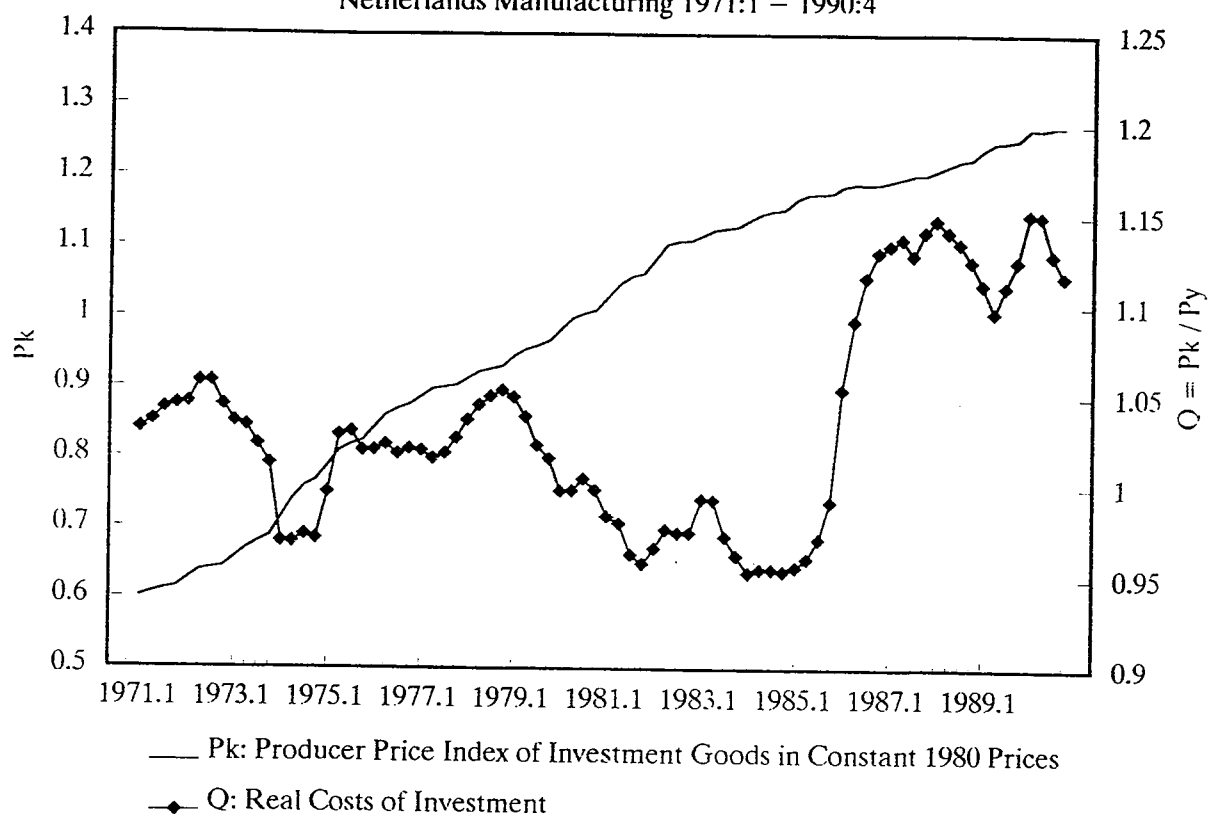
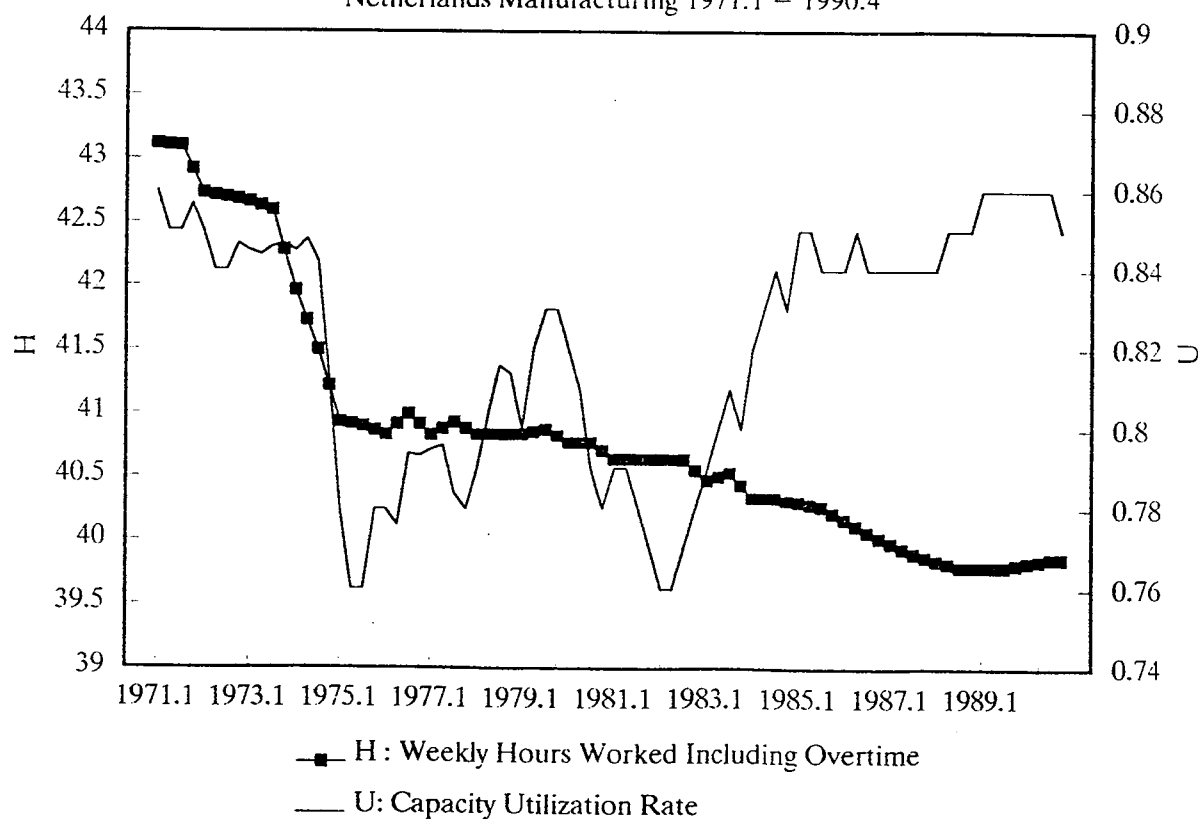


Fig A6: Production Factor Utilization

Netherlands Manufacturing 1971:1 – 1990:4



FOOTNOTES

- 1) See Kodde, Palm and Pfann (1990), Peeters, Palm and Pfann (1992), and Pfann and Palm (1993).
- 2) We find that $\ln A = -\frac{1}{T} \sum_{t=1}^T \ln(\phi_t) = -\frac{1}{T} \sum_{t=1}^T \phi_1(\ln Y_t^A - \mu_t)$,
where T is the sample size.
- 3) For details on the structural composition of the parameters in terms of technology parameters we refer to Kodde et al. (1990).
- 4) The list of instruments used is as follows : a constant term, ΔY_{t-2} , LIS_{t-3} , SIR_{t-3} , ΔL_{t-2} , ΔK_{t-2} , ΔW_{t-2} , ΔQ_{t-2} , where LIR and SIR are the long-term and the short-term interest rates respectively. The p-value of the test of overidentifying restrictions of the instrumental variables is .699.
- 5) The p-value of the WALD-statistic testing the diagonality of Λ yields .000.