

**Capital Accumulation as an Evolving
Competitive - Cooperative System**

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*Abstract**

The simplified Goodwin-like model of fluctuating growth is extended to allow for the effects of composition of capital upon real wage, thus augmenting the key relationship - the real Phillips curve. This development has been prompted by the Valtukh information-value hypothesis. Interconnections between a labour bill share in a national income, a rate of employment and a capital-output-ratio are represented by a three-dimensional competitive-cooperative system of ordinary differential equations. Typically, an increase in the labour income share enhances the capital-output-ratio which, in turn, adversely affects the employment rate. This model is capable of generating a cycle for these relationships.

The necessary and sufficient conditions for a local stability in the neighborhood of a nontrivial equilibrium are explicitly formulated. Stabilizing influence of the special form of technological progress on economic dynamics and destabilizing consequences of an excessive growth of the wage rate are displayed via computer simulations. In line with previous research (Ryzhenkov, 1993), this paper also outlines macroeconomic effects of delays in bargaining over automating/wage/employment/timing of innovation (separately and simultaneously) on the fluctuating economic growth. It is shown that irrationality or myopic rationality in bargaining, disregarding regularities of the whole system, may be detrimental.

The above deterministic descriptions avoid microeconomic aspect. Another reference model (Valtukh, Pusep, 1988) more explicitly mimics some important interconnections between technological and socio-economic sides of the capitalist mode of production and provides a framework for gaming.

Participants of the gaming experiments not only reveal information embodied in the initial conditions and rules of the game (memory), but also create/store new (synergetic) information via decision-making. In particular, they keep in mind the choices of techniques and give preference to the definite connections between different technologies among a broader set of possibilities. In addition, collective learning and teaching provide a new semantic level of information and higher economic efficiency. The key organisational feature of the economy in the information age, namely complementarity of competition, cooperation and coordination, becomes notably visual.

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1. Introduction

The experts of the OECD have recently concluded: "...technological progress has become virtually synonymous with long-run economic growth and increases in productivity... A large research agenda lies ahead of us in defining, measuring, interpreting, and understanding not only productivity growth and technology change but also how they are related to each other. OECD can take the leadership in marshalling the necessary financial and intellectual resources to achieve significant and much needed improvements in this crucial area of far reaching policy importance" (OECD, 1991: 133, 135).

Robert Reich calls in his influential book "The Work of Nations" for a national recommitment to the productivity and competitiveness of all nation's citizens. It is the skills and capacities of the people that have become the primary assets, and that must become the focus of social scientists' attention. "The skills of a nation's work force and the quality of its infrastructure are what makes it unique and uniquely attractive in the world economy" (Reich, 1991: 264). The complexity of modern technology and the large size of firms require firm-specific on-the-job training, internal promotion ladders and a stable and loyal work force.

According to the general living systems theory, evolution of societies depends primarily upon accumulation and transmission of learned information. Despite the overall trend, evolution does not in every case progress in the direction of greater complexity, adaptability, self-regulation, and social integration. In fact, collective learning processes can also proceed in pathological forms (see Miller, 1986: 428-440).

The requirements of the successful transformation in the former USSR are far from being satisfied: the dismantled central planning apparatus has not been replaced by an effective market framework; the majority of big state enterprises has not been restructured for operation in the new environment and new financial institutions are rather rudimentary et cetera.

The recent developments have been partially anticipated at Novosibirsk State University in 1989-1991 in gaming experiments with the model of a capitalist economy. The students unexperienced in the functioning of a capitalist economy did not accomplish effective economic growth and production was characterised by very deep disproportions and by being very prodigal with its human and material resources.

More experienced students modified not only strategies but very norms of behaviour by transiting from single-loop to double-loop learning (cf. Argyris, Schön, 1978: 18-29). They extended production by resolving incompatible norms and setting new priorities with associated strategies of cooperative behaviour. Social and individual interests drew together consequently (Ryzhenkov, 1990: 948-962). These experiments illustrate the irreversibility of economic development and show a tendency towards a higher level of (self-)organization as shown by some indicators of efficiency and effectiveness.

Goodwin's growth cycle model (1972) reflects contradictions in process of income distribution as a cycle-generating factor. This model has been analysed and extended in different ways. The present paper, along the same channel of investigation, focuses on issues of stability, fluctuations and long run trends in relation to improving skills and growing capital intensity. It reveals pitfalls of involuntary learning in social bargaining and illustrates the improvement in the functioning of the model economy due to voluntary learning.

2. The Simplified Glombowski-Krüger Model

The following most important assumptions are made:

- (1) two social classes (capitalists and workers);
- (2) only two factors of production, labour force and means of production, both homogenous and non-specific;
- (3) only one good is produced for consumption and investment purposes, the price is identically one;
- (4) production (supply) equals effective demand;
- (5) all productive capacities are operated;
- (6) all wages consumed, all profits saved and invested;
- (7) steady growth in the labour force;
- (8) real wage rate rises in the neighbourhood of full employment;
- (9) a constant flow of invention and innovation over time;
- (10) a constant labour intensity;
- (11) a qualification of the labour force corresponds to technological requirements.

The model omits Goodwin's assumption of constant capital-output ratio, but preserves his premise of the passivity of final demand.

The model is formulated in continuous time. Time derivatives are denoted by a dot, while growth rates are indicated by a hat.

The simplified version of the model consists of the following equations:

$$P = K/\sigma \quad (2.1)$$

$$a = P/L \quad (2.2)$$

$$u = w/a \quad (2.3)$$

$$\hat{a} = \mu_1 + \mu_2(\hat{K}/L), \quad 0 < \mu_2 \quad (2.4)$$

$$(\hat{K}/L) = v_1 + v_2 u, \quad v_1 \geq 0, v_2 > 0 \quad (2.5)$$

$$v = L/N \quad (2.6)$$

$$N = N_0 e^{nt}, \quad n = \text{const}, N_0 = \text{const} > 0 \quad (2.7)$$

$$w = -\gamma_1 + \rho v, \quad \gamma_1 > 0, \quad \rho > 0 \quad (2.8)$$

$$\Pi = (1 - w/a)P \quad (2.9)$$

$$\dot{K} = (1 - w/a)P. \quad (2.10)$$

Equation (2.1) postulates a technical relation between the capital stock (K) and net output (P). The variable σ is called capital-output-ratio. (2.2) relates labour productivity (a), net output (P) and labour input or employment (L). Equations (2.3) and (2.6) describe the shares of labour in national income (u) and the rate of employment (v), respectively. Equations (2.9) and (2.10) show that profit (Π), savings, investment and incremental capital ($\dot{K}$) are equal. Workers do not save at all. Investment lag is set aside.

Equation (2.4) is a linear form of Kaldor's technical progress function: the growth rate of labour productivity is assumed to depend linearly on the growth rate of capital intensity. "The use of more capital per worker... inevitably entails the introduction of superior techniques which require "inventiveness" of some kind... On the other hand, most...technical innovations which are capable of raising productivity of labour require the use of more capital per man - more elaborate equipment..." (Kaldor, 1957: 595). The Kaldor hypothesis of a constant flow of invention and innovation over time underlies the assumption of a technical progress function with the constant parameters. Although bunching or clustering of innovations is not assumed, the model is capable of generating long waves (see section 7).

Equation (2.7) defines the exponential growth of the labour supply (N) with the rate n. Equation (2.8) represents the linear approximation of the real Phillips curve. A rising rate of employment is assumed to affect wage increases (in real terms). There is no money illusion.

A mechanisation function is introduced in (2.5). It relates the growth rate of capital per employee (of capital intensity) to income distribution. "Mechanisation is encouraged by a high wage share, i.e., high labour costs per unit of net product" (Glombowski, Krüger, 1984: 265). (See also Fig. 1.)

3. Implications of Valtukh's Information Value Hypothesis

It is well known that Karl Marx left unsolved the problem of reduction of a qualified labour to a simple labour in his theory of commodity and surplus value. According to the information value hypothesis, the value of labour power of a particular qualification group (p_λ , $\lambda \in \Lambda$ - a set of qualification groups in the economy) and the amount of its direct labour (l_λ) are connected by the following relationship:

$$p_\lambda = \alpha_\lambda \psi_\lambda l_\lambda, \quad \text{where} \quad (3.1)$$

ψ_λ is the coefficient of reduction of qualified labour to simple labour, α_λ is the share of the workers in value added,

$$\begin{aligned} \alpha_\lambda &> 0, \quad \forall \lambda \in \Lambda, \\ \alpha_\lambda &= f(\psi_\lambda), \quad d\alpha_\lambda / d\psi_\lambda < 0, \quad \text{and} \\ dp_\lambda / d\psi_\lambda &> 0. \end{aligned}$$

The hypothesis evinces that the rate of surplus value, other things being equal, grows along with qualification. Real wage changes at a slower pace than the corresponding qualification (see Fig. 2 and 3) .

The higher the qualification, the higher is the capital intensity. The latter may be used as an indicator of qualification (Valtikh, 1991: 21-44). This hypothesis will help us to extend the model of capital accumulation.

Instead of assuming, as in the usual Phillips relation, that the rate of change of the wage rate (w) depends only on the employment rate (v), let this rate be additionally influenced by the rate of change of capital intensity (K/L):

$$\hat{w} = -\gamma_1 + \rho v + \gamma_2 + \beta(\hat{K/L}), \quad \gamma_2 \geq 0, \quad \beta > 0, \quad \beta < \mu_2. \quad (3.2)$$

This modification also takes into consideration the historical or moral element in the value of labour power. The linear approximation is tentative and perhaps requires more precise definitions. Equation (3.2) will be used in obtaining a convenient statement of the model.

The diffusion of lean production in the industrialised countries based on re-engineering, training employees in multiple skills and pushing decision-making authority as far down in an organisation as possible empirically supports these arguments (see Ehrbar, Roth 1993). Furthermore, this upgrading tendency obviously overweighs a counter-tendency. The latter may prevail in some branches or even in an economy as a whole at some periods of historical development (then $\beta < 0$). Please note that we are abstracting from the institutional arrangements for the supply of skills.

4. A Nontrivial Equilibrium in the Modified Model

The central variables of the modified model are the employment rate (v), the labour bill share (u) and the capital coefficient (σ). Let us first consider the evolution of the latter. Substitution and logarithmic differentiation yield:

$$\begin{aligned} \hat{\sigma} &= (\hat{K/P}) = (\hat{K/L}) - (\hat{P/L}) = -\mu_1 - \mu_2(\hat{K/L}) + (\hat{K/L}) = \\ &= -\mu_1 + (1 - \mu_2)(v_1 + v_2 u). \end{aligned} \quad (4.1)$$

The employment rate (v) is defined by the equations:

$$\hat{v} = (\hat{L/N}) = \hat{P} - \hat{a} - \hat{N} = (\hat{P/K}) + \hat{K} - \hat{a} - \hat{N} = -\hat{\sigma} + (1 - w/a)/\sigma - \hat{a} - n. \quad (4.2)$$

We have used in (4.2) $\hat{K} = (1 - w/a)P/K = (1 - w/a)/\sigma$. Substitution and transformation allow us to write (4.2) in a simpler form:

$$\hat{v} = (1 - u)/\sigma - (v_1 + v_2 u) - n. \quad (4.3)$$

The labour bill share (u) changes according to

$$\begin{aligned} \dot{u} = (\dot{w}/a) = \dot{\hat{w}} - \dot{\hat{a}} = -\gamma + \rho v + \beta(v_1 + v_2 u) - (\mu_1 + \mu_2(v_1 + v_2 u)) = \\ -\gamma + \rho v - \mu_1 + (\beta - \mu_2)(v_1 + v_2 u), \text{ where } \gamma = \gamma_1 - \gamma_2. \end{aligned} \quad (4.4)$$

Thus, we have the following system of ordinary differential equations:

$$\dot{\sigma} = -(\mu_1 + (\mu_2 - 1)(v_1 + v_2 u))\sigma \quad (4.5)$$

$$\dot{v} = ((1 - u)/\sigma - (v_1 + v_2 u) - n)v \quad (4.6)$$

$$\dot{u} = (-\gamma + \rho v - \mu_1 + (\beta - \mu_2)(v_1 + v_2 u))u. \quad (4.7)$$

The singular points (equilibrium state) are given by:

$$\begin{aligned} E_1 = (\sigma_1, v_1, u_1) = (0, 0, 0) \text{ and} \\ E_2 = (\sigma_2, v_2, u_2), \text{ where} \end{aligned} \quad (4.8)$$

$$u_2 = \mu_1/(v_2(1 - \mu_2)) - v_1/v_2, \quad \sigma_2 = (1 - u_2)/(v_1 + v_2 u_2 + n) \text{ and} \\ v_2 = (\gamma + \mu_1 + (\mu_2 - \beta)(v_1 + v_2 u_2))/\rho.$$

(σ_2 follows from (4.6), v_2 - from (4.7) and u_2 - from (4.5).)

The three-dimensional system is obviously nonlinear and could, potentially, display a very rich spectrum of behaviour.

If $\mu_1 = 0$ and $\mu_2 = 1$ then $\dot{\sigma} = 0$ and the capital-output-ratio is determined by the initial condition ($\sigma = \sigma_0 > 0$). The system degenerates into the two-dimensional system:

$$\dot{v} = ((1 - u)/\sigma_0 - (v_1 + v_2 u) - n)v \quad (4.6a)$$

$$\dot{u} = (-\gamma + \rho v + (\beta - 1)(v_1 + v_2 u))u. \quad (4.7a)$$

The nontrivial equilibrium in this case is given by (4.8a):

$$\begin{aligned} u_e = (1 - \sigma_0(v_1 + n))/(v_2 \sigma_0 + 1), \\ v_e = (\gamma + (1 - \beta)(v_1 + v_2 u_e))/\rho. \end{aligned} \quad (4.8a)$$

We will provide an example of a long wave for this case in section 7.

The Jacobian of the model evaluated at the nontrivial equilibrium is given by

$$J = \begin{vmatrix} 0 & 0 & v_2(1-\mu_2)\sigma_2 \\ -v_2(1-u_2)/\sigma_2^2 & 0 & -v_2/\sigma_2 - v_2v_2 \\ 0 & \rho u_2 & (\beta - \mu_2)v_2u_2 \end{vmatrix}.$$

Thus, the characteristic polynomial is $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$, where

$$a_0 = -\det(J) = (v_2(1-u_2)\rho u_2 v_2(1-\mu_2))/\sigma_2 > 0$$

$$a_1 = \rho u_2 v_2(1/\sigma_2 + v_2) > 0$$

$$a_2 = -\text{trace}(J) = (\mu_2 - \beta)v_2u_2 > 0, \text{ if } \mu_2 > \beta.$$

The Routh-Hourwitz conditions are necessary and sufficient for local stability and require that $a_0 > 0$, $a_1 > 0$ and $a_1a_2 > a_0$. The first and second inequalities are satisfied, whereas the third inequality corresponds to (4.9):

$$(v_2\sigma_2 + 1)(\mu_2 - \beta) > ((1-u_2)/u_2)(1-\mu_2) > 0. \quad (4.9)$$

Provided that the inequality (4.9) holds, the dynamics of the modified model (4.5), (4.6) and (4.7), in the neighborhood of its equilibrium, are Poincaré (locally) stable (cf. van der Ploeg, 1987). Note that in (4.9), $(1-u_2)/u_2$ is equal to the rate of surplus value estimated at the equilibrium.

The inequality (4.9) is not true, if $\mu_2 < \beta$. So $\mu_2 > \beta$ is a necessary condition of the local stability. The presence of coefficient $v_2 > 0$ on the left side of (4.9) shows that a distribution-induced change in the speed of technological progress produces a stabilizing influence. The result gives support to the view expressed in section 3.

5. Trend in the Modelling Economy

Extended reproduction along the steady state path displays the following properties. Output grows at its "natural rate", the sum of labour supply growth and productivity growth:

$$\hat{P}_2 = \hat{a}_2 + n = \mu_1/(1-\mu_2) + n.$$

The growth of labour productivity and real wage is at the rate

$$\begin{aligned} \hat{a}_2 = \hat{w}_2 &= \mu_1/(1-\mu_2), \text{ if } \mu_1 \neq 0 \text{ and } \mu_2 \neq 1, \\ \hat{a}_e = \hat{w}_e &= v_1 + v_2u_e \text{ (see (4.8a) above), if } \mu_1 = 0 \text{ and } \mu_2 = 1, \end{aligned}$$

whereas the profit share, the capital-output ratio and, therefore, the rate of profit itself are constant. Capital stock increases at the rate $\hat{K}_2 = \hat{P}_2$ ($\hat{K}_e = \hat{P}_e$) which is equal to the profit rate

$\Pi_2/K_2 = \hat{P}_2 = \hat{K}_2$ (or $\Pi_e/K_e = \hat{K}_e = \hat{P}_e$ in the degenerated case).

The labour income share, the rate of employment and capital-output-ratio are constant.

P. Romer (1989) reproduces the Kaldor stylized facts and provides other prominent features of the data that characterize the process of economic growth in advanced economies.

There is a good agreement between the properties of the steady-state growth in our model and Kaldor's five stylized facts:

1. Output per worker shows continuing growth with no tendency for a falling rate of growth of productivity.
2. Capital per worker shows continuing growth.
3. The rate of return on capital is steady.
4. The capital-output ratio is steady.
5. Labor and capital receive constant shares of total income.

It will be rather simple to reflect in a similar model such a prominent feature of the data as a negative correlation of population growth rates with the level of income, mentioned by P. Romer. This and other empirical regularities, outlined in his interesting paper, cannot be analyzed here.

Similar to Kaldor (1957), we have determined the shares of wages and profits as well as the rate of profit on capital quite independently of the principles of marginal productivity. The rate of profit on capital depends on the coefficients of the technical progress function which, in turn, determine the rate of growth of labour productivity. Unlike Kaldor (1957), our model typically yields the persistence of unemployment.

6. A Deeper Analysis of Mathematical Properties of the Goodwin-like Model

We will now place this model in a nonlinear cooperative-competitive network. A differential equation $\dot{x} = f(x)$ defined on $G \subset \mathbb{R}^n$ is said to be *cooperative* if $\partial f_i(x)/\partial x_j \geq 0$ for all $x \in G$ and all $i \neq j$ (the growth of every component is enhanced by an increase in any other component). *Competitive* systems are defined by $\partial f_i(x)/\partial x_j \leq 0$ for all $i \neq j$ (Hofbauer, Sigmund, 1988: 158). It may be easily shown that the system (4.5) - (4.7) represents a mixed case, i.e., a *competitive-cooperative* system:

$$\frac{\partial f_1(\sigma)}{\partial v} = 0, \quad \frac{\partial f_1(\sigma)}{\partial u} = (1-\mu_2)v_2\sigma \quad \begin{cases} \leq 0, & \text{if } \mu_2 \geq 1 \\ > 0, & \text{if } \mu_2 < 1 \end{cases} \quad (6.1)$$

$$\frac{\partial f_2(v)}{\partial \sigma} = -(1-u)v/\sigma^2 < 0, \quad \frac{\partial f_2(v)}{\partial u} = -v/\sigma - v_2v < 0 \quad (6.2)$$

$$\frac{\partial f_3(u)}{\partial \sigma} = 0, \quad \frac{\partial f_3(u)}{\partial v} = pu > 0. \quad (6.3)$$

If $\mu_2 < 1$, then the increasing labour income share enhances the capital-output-ratio which adversely affects the employment rate. This model is capable of generating a cycle for these relationships.

Unfortunately, no global theorems have yet been proved about these mixed competitive-cooperative feedback networks (Grossberg, 1988: 38). In the three-and-higher-dimensional case the Poincaré-Bendixson theorem cannot be applied. The Hopf theorem may constitute the only tool to establish the existence of closed orbits.

In this study we choose β (see the equation (3.2)) as a bifurcation (control) parameter, although it is possible to select either.

The characteristic property of a dissipative system implies that the volume of an element of the phase space shrinks to zero as time progresses, when trajectories approach an attractor (see Fig. 2.15, Lorenz, 1989: 56). Notice that the majority of dynamical systems in economics belongs to the class of dissipative systems. Formally, this property can be examined with the help of the Lie derivative or the divergence defined as

$$\dot{V}/V = \text{div}(f) = \sum \partial f_i / \partial x_i, \quad i = 1, \dots, n, \quad (6.4)$$

with V as the "volume" and $\text{div}(f)$ as the divergence of f . The Lie derivative is negative when the system is dissipative (see Lorenz, 1989: 55-56). One of the important properties of a dissipative system is time irreversibility on transients to attractors (Lorenz, 1989: 63).

In our model the Lie derivative is calculated as follows:

$$\begin{aligned} \dot{V}/V &= (-\mu_1 - (\mu_2 - 1)(v_1 + v_2 u)) + ((1 - u)/\sigma - (v_1 + v_2 u) - n) + \\ &+ (-\gamma + \rho v - \mu_1 + v_1(\beta - \mu_2) + 2(\beta - \mu_2)v_2 u) = \\ &= -2\mu_1 + (v_1 + v_2 u)(-2\mu_2 + \beta) + (\beta - \mu_2)v_2 u + (1 - u)/\sigma - v - \gamma + \rho v. \end{aligned} \quad (6.5)$$

Let us now consider the local behaviour of the solution of

$$\dot{x} = f(x) \quad (6.6)$$

near the point $E_2 (x_2 = (\sigma_2, v_2, u_2))$ in R^3 .

The linear equation

$$\dot{y} = Jy \quad (6.7)$$

can be solved explicitly.

The following constellation of coefficients has been chosen:

$$\mu_1 = 0.12, \mu_2 = 0.8, v_1 = 0.12, v_2 = 1.5, \rho = 1.8, \beta = 0.2, \gamma = 1, n = 0.02.^1$$

The requirement of local stability (4.9) has not been infringed. We have found that x_2 is hyperbolic in the sense that all eigenvalues of J have nonvanishing real parts. This allows us to apply the theorem of Hartman and Grobman (Steeb, Kunick, 1989: 39-40), which states that both systems (6.6) and (6.7) are topologically equivalent in a sufficiently small neighbourhood of the fixed point (E_2). It gives rise to the following equilibrium values of our most important variables:

$$\sigma_2 = 1.0968, u_2 = 0.32, v_2 = 0.822.$$

Because the real parts of the eigenvalues are all negative, the equilibrium E_2 is a sink. Moreover, it is locally asymptotically stable (cf. Hofbauer, Sigmund, 1988: 54). The original constellation of coefficients in the model yields the negative Lie derivative (see Fig. 4) illustrating the smaller distance between the orbits.

We have shown that the system (6.6) possesses an attractor in the form of the fixed point. Further investigations are necessary to answer whether this system owns attractors in the form of limit cycles, orbits in the phase space, or in the form of strange objects.

The inequality (4.9) turns into equality if $\beta_0 = 0.63933$. The system

$$\dot{x} = f(x, \beta), \quad (6.8)$$

where $x \in \mathbb{R}^n$, $\beta \in \mathbb{R}$, has an equilibrium (x^*, β_0) at which the following properties are satisfied.

The Jakobian of (6.8) has one pair of pure imaginary eigenvalues $(\bar{a} \pm bi)$ and no other eigenvalues with zero real parts (moreover, at the point $\beta = \beta_0$ $\delta \bar{a} / \delta \beta > 0$).

According to the Hopf bifurcation theorem (an existence part), there exist some periodic solutions bifurcating from $x^*(\beta_0)$ at $\beta = \beta_0$ and the period of the solutions is close to $2\pi/b$ (see Table 1). When β is increased from $\beta < \beta_0$ to $\beta > \beta_0$, the single fixed point changes its stability because the real part ($\bar{a}$) becomes positive. This theorem establishes only the existence of closed orbits in a neighborhood of x^* at $\beta = \beta_0$, and it does not clarify the stability of orbits, which may arise on either side of β_0 . The stability properties of the closed orbits depend on the nonlinear terms because in the Hopf bifurcation the real parts of the eigenvalues of Jacobian, i.e., of the linear approximation, vanish.

Further increasing of β (at $\beta = 0.73$) provides us with one real negative eigenvalue and a conjugate pair of eigenvalues with positive real components. It means that the corresponding fixed point is the saddle (see Table 1). The sign of the Lie derivative periodically changes, if $\beta = \mu_2 = 0.8$, i.e., when β is higher than the threshold (β_0). At $\beta = 0.82$ there is the saddle-focus (cf. Wiggins, 1990: 553).

Our computer simulations have shown that the initial system of the ordinary differential equations seems to be more stable than its linear approximation in the neighborhood of the equilibrium under the above constellation of coefficients and the chosen initial magnitudes of the main parameters u , v and σ (the non-linear system exhibits converging fluctuations even if β is greater than β_0 but smaller than 0.73).

Using the Wolf algorithm (see Wolf, 1986) we have calculated the Lyapunov exponents for a smaller magnitude of μ_2 for $\beta_0 = 0.63933$ (see Fig. 5). These exponents hint at deterministic chaos and at an existence of a strange attractor. This result is rather unstable with respect to parameter choices. We will examine this aspect more closely in further investigations.

7. Long Waves and Other Nonharmonic Fluctuations

The model is not only able to incorporate growth, but may also reflect the persistence of cycles. To illustrate this cyclical growth it is necessary to choose some appropriate values of coefficients and initial magnitudes of variables. We will provide an example of damping cycles by using the very first constellation of coefficients.

If we set the initial magnitude of the rate of employment $v_0 = 0.722 \neq v_2$ without an initial displacement of other variables from their equilibrium values ($u_0 = u_2$, $\sigma_0 = \sigma_2$), a damping cyclical motion is obtained, the length of the cycle being approximately 6 years.²

The labour bill share peaks at a value of 0.36 at year 4.4 and 0.34 at year 10.4. The damping ratio defined relative to the mean point of the oscillation is, therefore, approximately

$$1 - (0.34 - 0.32)/(0.36 - 0.32) = 0.5 \text{ (cf. Mass, 1975: 52).}$$

The fluctuations are not strictly periodic. The amplitude and phasing of each variable are determined structurally (cf. in a linear case all variables oscillate with the same frequency and damping; only their amplitudes and phasing differ, these being parameters fixed for each variable separately by extraneous factors or initial conditions. See Allen, 1954: 161).

The above values of variables are far from being realistic. The following values better calibrate the model to historically reasonable magnitudes (see Tabl. 2). In spite of the Kaldor assumption of constant flow of invention and innovation over time applied, our model generates long waves. To test whether this result is an artefact of the specific constellation of the parameters, we have run the model for the relevant points in parameter space. Table 2 and Fig 6-8 summarize these experiments and display the relative speed of convergence for different magnitudes of the two significant parameters. Our results may be interpreted as supporting the conclusion that "no intrinsic clustering

of innovations is necessary to produce long period fluctuations of economic activity" (Silverberg, Lehnert, 1992: 17).

8. Consequences of Bargaining Delays

According to John Sculley, the modern reorganisation of work could prove as massive and wrenching as the Industrial Revolution. We agree with the experts who warn that these changes can only arise with social frictions (see Ehrbar, Roth, 1993).

Now suppose the behaviour of the modelling system is influenced by labour contract terms and consider an elicited response to an introduction of wage lag in the modified Phillips equation (3.2), "caused by the extension of collective bargaining and particularly by the growth of arbitration and conciliation procedures" (Phillips, 1958: 293). Notice that the anticipations of social tensions may lead to an abnormal increase in real wages, but we will not take this aspect into account.

C. Chiarella has introduced a time lag into the wage formation equation of the original Goodwin model and by use of the centre-manifold theory has shown that the resulting system of three differential equations displays the stable limit cycle behaviour at least for a relatively small time lag (Chiarella, 1990: 70-80). However, the dynamic properties in the model for larger values of the time lag have not yet been investigated.

J. Forrester writes: "The effects of a delay depend on where it is in the system. Delays may produce either amplification or attenuation of disturbances depending on where they appear" (Forrester, 1969: 349).

We will address this problem in a broader context of social bargaining by applying a case study approach. Being mostly interested in macroeconomic consequences of a duration of bargaining we will treat the bargaining as a "black box" at a rather high level of abstraction.

The investigation consists of a number of case studies. Four of the case studies consider elementary bargaining over automating/wage/employment/timing of innovation (separately), and the fifth case study analyses simultaneous bargaining over all of these issues. In numerical simulations we have used the first-order and the third-order exponential delays, represented by the DYNAMO built-in functions SMOOTH and DLINF3, respectively.

The third-order delay is equivalent to 3 cascaded single-order delays, with each single-order delay having a delay time of $DEL/3$, where DEL is the same total average delay (Forrester, 1969: 86-92). The transient responses to changes in flow rate usually differ.

In particular, wage change will be related not to the current employment ratio, but to the ratio lagged, on average, by several months. An explosive cyclical motion around the

trend may be obtained, for example, by $DEL = 0.25$ (three months) in both cases (under the same constellations of the coefficients and initial values as in the example of the 6-years cycle above). The amplitude of oscillations grows faster if the model is extended by the third order delay. As a result of introduction of the delay, the system becomes considerably less stable. This sensitivity shows that wage lag is one of the most crucial parameters in determining the system's dynamical performance.

A great deal of work is necessary for modelling international differences in bargaining systems (in wage setting behaviour, in particular) in order to shorten the gap between complex industrial relation systems and the rather stylized characterizations employed in this paper.

Table 3 and Fig. 9-14 summarize the results of our case studies. We observe that delays which do not harm stability in elementary bargaining, under an illustrative constellation of the parameters and the initial conditions of the model, may violate stability in simultaneous bargaining. In order to guarantee stability, the latter should be less lengthy than the former.

A conflict may be an unintended or intended consequence of the overextended bargaining. As we have seen, under the conditions, explicitly formulated, the dynamics of the model are locally stable in the neighborhood of its equilibrium. These dynamics are not robust with respect to the definite changes in social relations. The society only capable of involuntary learning is condemned to experience escalating conflicts, as has been illustrated. One should keep in mind, however, that growing systems usually develop in the direction of more elaborated adjustment processes (Miller, 1978: 108-109).

Notice that an introduction of the information lags not only raises a dimensionality of the system of the differential equations but alters the Routh-Hourwitz conditions of local stability as well. The non-trivial equilibrium of the system is, of course, also an equilibrium of the similar system with a higher dimensionality, but at present stage of investigation we are not considering its stability properties.³

9. Competition, Cooperation and Diffusion of Innovations

The above deterministic descriptions avoid an explicit treatment of the discrete stochastic character of innovations at the microlevel. So it would be worthy to expose another facet of the evolutionary approach to capital accumulation and technical change by using an appropriate model.

Let us now consider diffusion of innovations adopted by private producers under conditions of market disequilibrium. Demand will be a relatively autonomous element of the capitalist reproduction. Production capacities will not necessarily be operated at full. Capitalists and workers are allowed to save. In order to exert additional evolutionary pressure on entrepreneurs, we are postulating a constant labour supply. We

will analyse voluntary learning within the class of capitalists. Our focus will be feedbacks between micro- and macrolevels. Thus, some questions, which have remained unanswered in previous parts of this paper, will be addressed in this section.

We will use the results of two computer-supported gaming experiments conducted at Novosibirsk State University that were based upon a simulation model of commodity production (Valtukh, Pusep, 1988) basing on (Valtukh, 1987). This model has been used in the classroom (Ryzhenkov, 1990, 1991a, 1991b, 1991c).

Note the most relevant assumptions. There are three monoprodukt sectors. Each kind of commodity can be produced by three or fewer technological modes. Every elementary technique can belong to different capitalists, but initially there are nine capitalists possessing nine firms with respective elementary techniques. Technological modes diffuse here endogenously through the expansion of the innovative firms and through imitations by other firms. Inventions and innovations occurred in the past not considered explicitly. The model does not take into account patenting and product innovations.

A capitalist is allowed to construct a "new" firm with any given technique. There is no problem if the capitalist has only one "old" firm. However, if the capitalist has two "old" firms, a creation of a "new" firm requires the giving up at least one "old" firm losing fixed assets. These conditions imply the administrative market-entry and market-exit restriction. Natural resources are not scarce and supply of labour power is fixed.

9.1. The First Gaming Experiment

Participants of the first game were students learning the economic theory. The most active of them also have taken part in the second experiment. The first gaming experiment encompasses nine years.

In each sector there is a dominant technology which has the greatest rate of profit. Dominated methods have been either eliminated or suppressed while dominant techniques have diffused within established sectors through new capital investment. The ranking of technological modes depending on their profitability is characterized by stability for each sector but not for the economy as a whole (see Table 4). Technique 3 is the most profitable in the first sector, technique 4 - in the second, and technique 8 - in the third sector.

A blank in the Table 5 means that the respective technique is not used in the year of account. Table 4 contains magnitudes for both kinds of technologies (applied and withdrawn) since all these data are relevant for the decision-making.

These choices of techniques bring about some inefficiency from the social point of view since the most profitable methods are not necessarily the most labour-saving. A ranking of techniques according to profitability diverges from their inverse ranking according to individual labour values (cf. Tables 4 and 5).

The dominant (third) technology is the most total labour-saving only in the first sector. Technologies 8 and 9 have close levels of profitability and total labour intensity as well. Their shares in gross output of the third sector oscillate for the whole period of the game; the most socially effective (ninth) technology has not won the contest.

Capitalists of the second sector have preferred the most profitable (fourth) technology to the most progressive (fifth) one removed. Fig. 15 graphs three measures of diffusion of the fourth technique inside the second sector producing elements of fixed assets and consumer goods. The discrete points have been connected by linear pieces for greater visuality. The picture reminds us of the well-known S-shaped form familiar to the researches of diffusion (cf. Mansfield, 1961; Silverberg, Dosi, Orsenigo, 1988). The similar S-shaped trajectories characterize the diffusion of the third technology in the first sector.

The players' investment projects are subject to trial and error. The bounded rationality of players is reflected in Fig.15 where the output share of the fourth technology declines in the second year. During nine years 45 investment projects are carried out. A quantity of engaged investment projects appears to be correlated with a technology profitability: Spearman's correlation coefficient (between final ranks) is equal to 0.59 (connections are disregarded). Note that investors (including new entrants) did not chose any technological mode with the lowest profitability inside a sector (modes 1, 6, 7) and that relative efficiency of technological mode is to a great extend invariant to changes in prices and values of commodities.

The general economic decline in the middle of the game forces players to develop the centralized crisis management (co-ordination costs have been disregarded). In particular, they have agreed to stimulate final demand hiring excessive labour power. This strategy of benevolent commodity producers seems to be successful, although it goes beyond capitalist social relations in their pure form.

9.2. The Second Gaming Experiment

The second case study illustrates how the players have improved the results. The game consists of 11 rounds. Although the initial conditions are the same, technical- and-economic efficiency is higher than in the previous case (cf. Table 8, Figures 15 and 16, 19 and 20). The players get in technologies and out of inferior ones faster and more effectively (timing, selection, obsolescence) than in the first game. In particular, they have gotten rid of technologies 1, 2, 6, 7 which hinder the economic growth (cf. Tables 4-7). The diffusion curves for technologies 3, 4, 8 are characterized by more rapid and severe increase during the early phases than in the previous experiment (for example, compare Figures 15 and 16).

It is my opinion, that the second game can be divided into two connected stages: the first one takes place during the years 1 to 6, the second one during the years 7 to 11. An

economic crisis in the sixth year establishes a border between these stages. At the end of the sixth round the players have decided to coordinate their behaviour closely in order to achieve reproduction on an increasing scale.

After some kind of Shumpeterian creative destruction, our players have used only the most socially effective technologies (3, 5, 8 and 9) by the end of the game (the year 11). It is interesting to note that the growth over time in the percentage of firms having introduced the fifth technology in combination with other two measures of adoption conforms to the S-shaped growth curve in spite of the decline in the first half of the game resulting from diffusion of the fourth technology. Both experiments show that the slower a sector grows, the slower is the rate of imitation.

Horizontal parts of the trajectories displayed at Fig.19 and 20 mean that resources for further diminishing of total labour intensities are almost exhausted. The adoption of the more effective technological system brings nearer the technological frontier. The need for new innovations and for their adoption has been perceived more acutely than ever before. It is noteworthy here to mention that Japan, being once the main rapid imitator, becomes the leader in a variety of innovations.

In agreement with the classical theory of value, relative prices finally drop for sectors with the higher rates of growth of labour productivity (see Table 8). Despite continuous modified reproduction of oligopolistic structures prices tend to move in the long run in accordance with labour values (see Ryzhenkov, 1991a).

It is known that while technological diffusion tends to wither away inter-firm asymmetries, the process of innovation keeps creating new ones (Dosi, 1986: 295). Although our experiments exaggerate the "entropic tendency" towards technological uniformity, they also contain a "negentropic tendency". The players not only reveal the information embodied in the initial conditions and rules (as participants of J. H. Conway's game "Life" do), but create new information. Their decision-making provides the higher economic order. They keep in mind the choices of techniques by giving preference to the definite connections between different technologies among the broader set of possibilities and generate new behavioural algorithms, thus raising the functional efficiency. Exporting information from outside (for example, teaching) accelerates the progressive evolution.

The players understand the necessity of more cooperative behavior in order to overcome the crisis and take advantage of the incurring of short-term losses in profitability for long-term gains. The deviation from the spontaneous market economy should be considered, from my point of view, as the particular manifestation of the tendency to a higher level of (self-)organization.

10. Conclusion

Players improve their technology management and attain some guidelines for allocating limited resources among investment projects.

In the current global environment the effective technology management is often called the hidden competitive advantage. Our gaming experiments are intended to assist students in making the management of technology more clear and transparent.

It might be reasonable to imagine that the successive experiments represent the development of two countries. The second least developed country took advantage by learning the methods of production which had been used in the first one. Therefore these experiments would appear to confirm the thesis that international learning represents the source of international gains (cf. Pasinetti, 1981: 259-271).

The main shortcoming of the above models, in my opinion, is the assumption of homogeneity of factors (labour force and means of production). I agree with the experts of the OECD, who believe, that in order to measure the effect of technological change on productivity growth more accurately, it is necessary to keep track of different vintages of fixed assets embodying different levels of knowledge, explicitly considering the link between the rate of technological progress and scrapping rates (cf. OECD, 1991: 137). The measurement of labour inputs should reflect changes over time in the average number of hours worked, and possibly capture qualitative attributes of the work force (education and skill levels).

Both models are based on the assumption that the qualification of labour force corresponds to technological requirements. The real processes of secular skill upgarding through education, training and learning-by-doing, on the one hand, and the obsolescence of skills acquired on the former phases of technological advance, on the other hand, should be explicitly reflected in the further research.

Voluntary learning which may alter structure, delays and amplifications should be incorporated in the Goodwin-like model(s) as well. With the assistance of the elaborated approach one can test the consequences of alternative modes of paying workers over the business cycle in "a wage economy" (considered above) and in "a share economy" where workers are paid in part via profit or revenue sharing. More interesting lag distribution can be explored, etc. According to Richard Goodwin, one could also profitably investigate, the effects of various types of policies pursued by workers, employers and state.⁴

Money supply, credit and the interest rate(s), money wage formation and price setting may be introduced as well. It is not yet clear whether a hybrid model of the business cycle(s) or a system of models interacting with each other would implement these suggestions. One of the prospective directions for further investigations would be the study of instability as a moment of evolution towards a higher socio-economic order.

Notes

1. It is not implied that this purely illustrative constellation is in fact empirically accurate.
2. Professor R. Goodwin has pointed out that the cycle length of 6 years should, if possible, be altered by parameters changes: it is too long for the short cycle and not long enough for the major cycle in capitalists systems.
3. Professor W. Weidlich has remarked upon the possibility of a transformation of the stable equilibrium in unstable equilibrium after the introduction of information lags in the initial system and advised me to investigate analytically this problem. I agree with this valuable observation.
4. I am grateful to Professor R. Goodwin for these hints.

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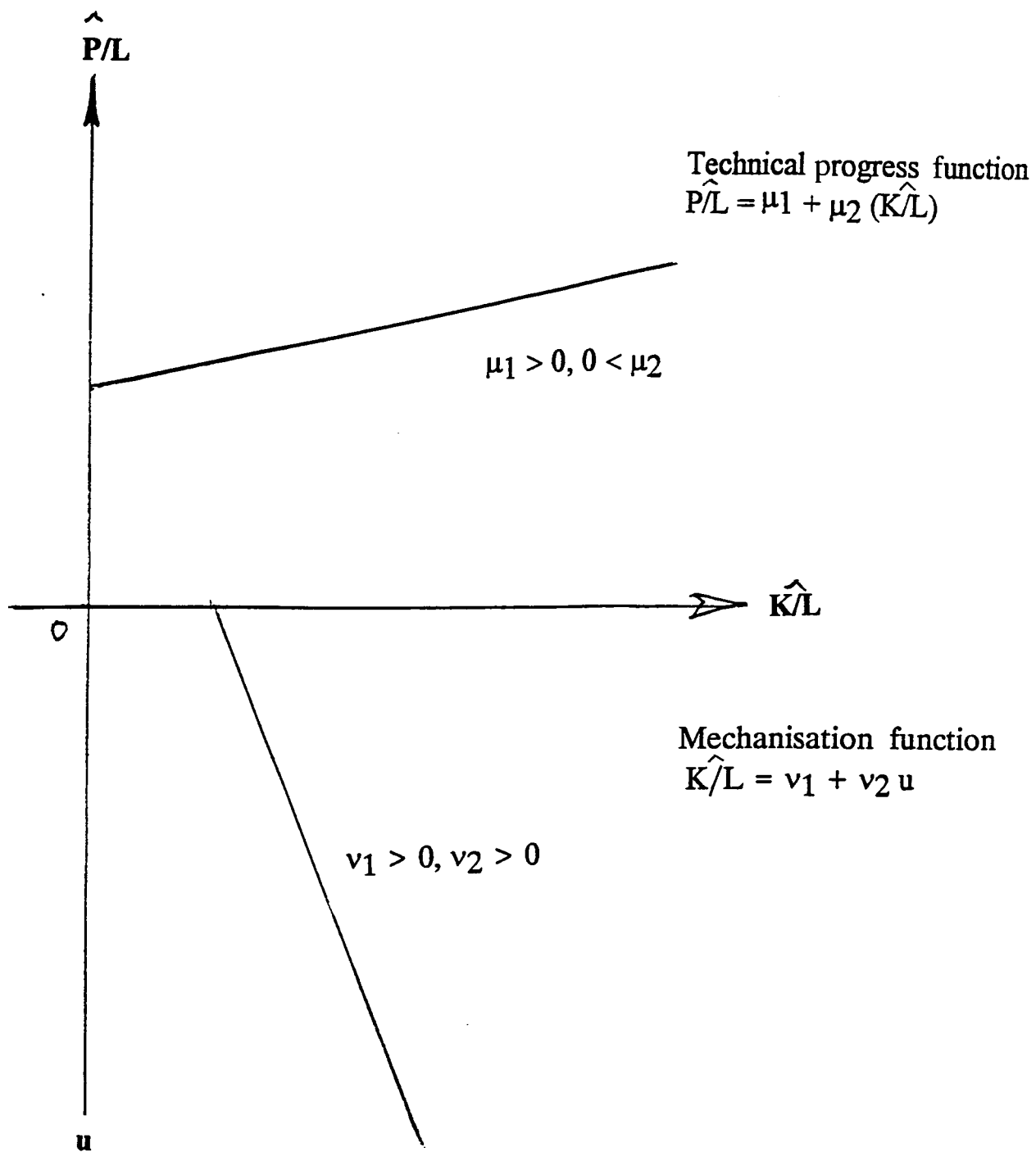


Figure 1. The linear approximations of the technical progress and mechanisation functions.

Adapted from: Glombowski, Krüger, 1984: 266.

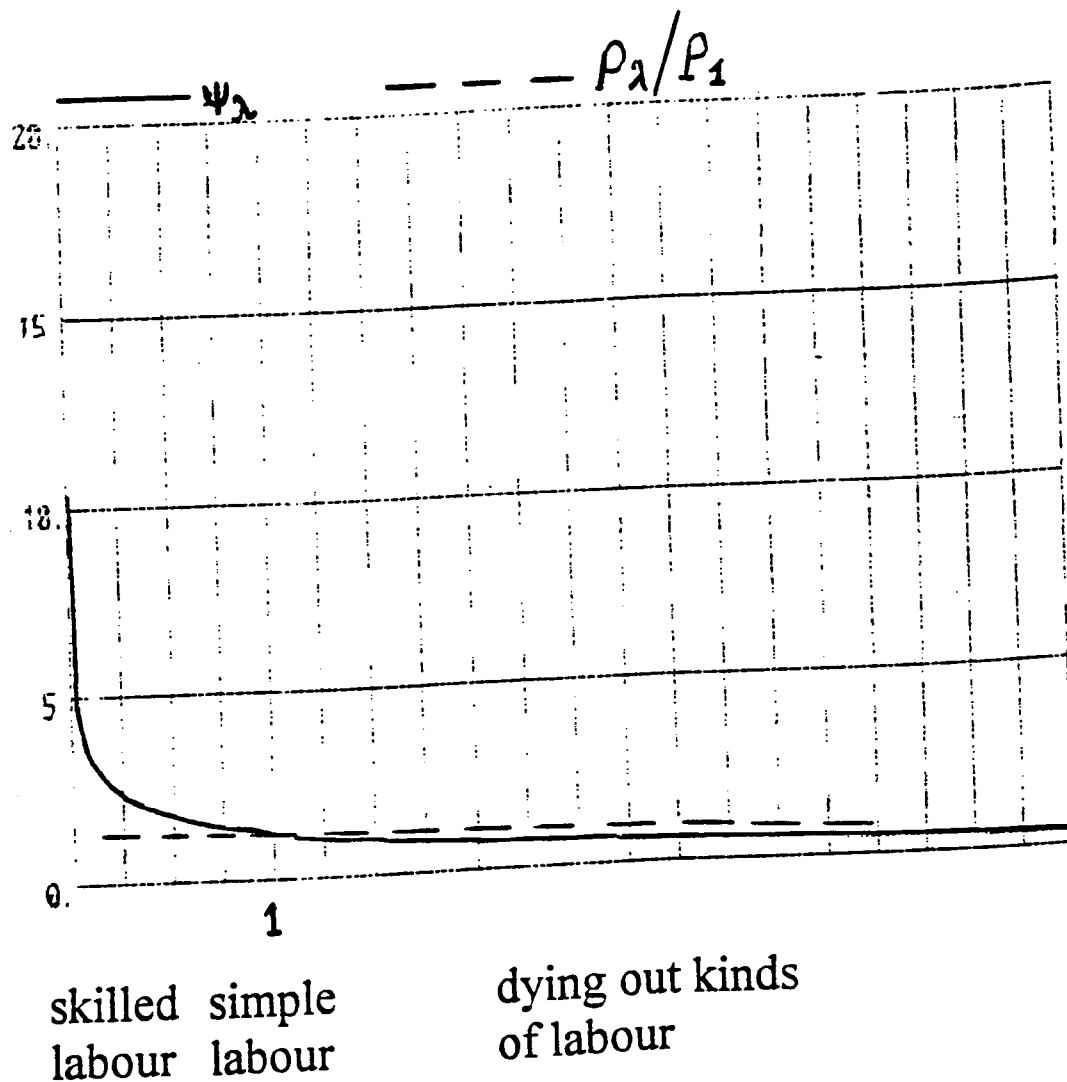


Figure 2. Coefficients of reduction of skilled labour to a simple one (ψ_λ) and ratios of wage rates (ρ_λ/ρ_1) for different qualification groups ($\lambda \in \Lambda$)

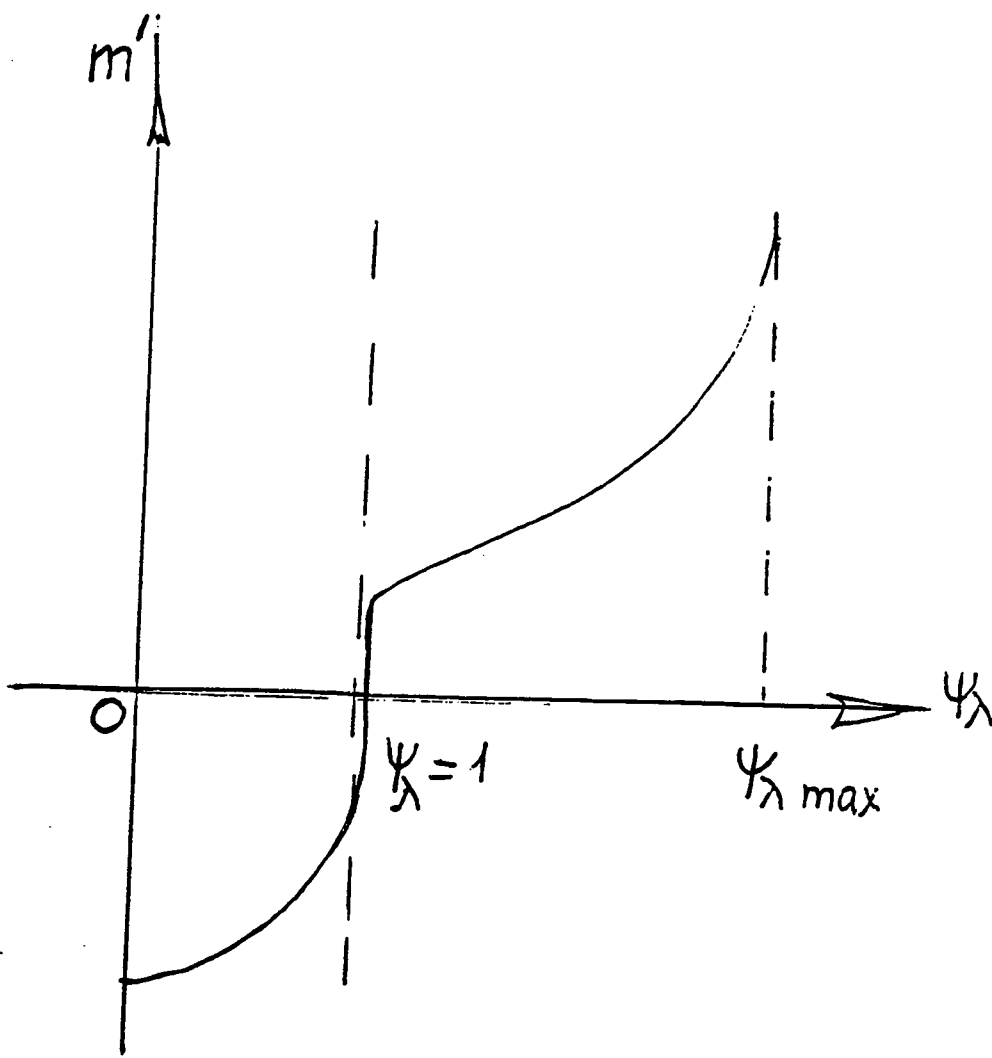


Figure 3. Rate of surplus value $m' = (1-u)/u$ versus qualification

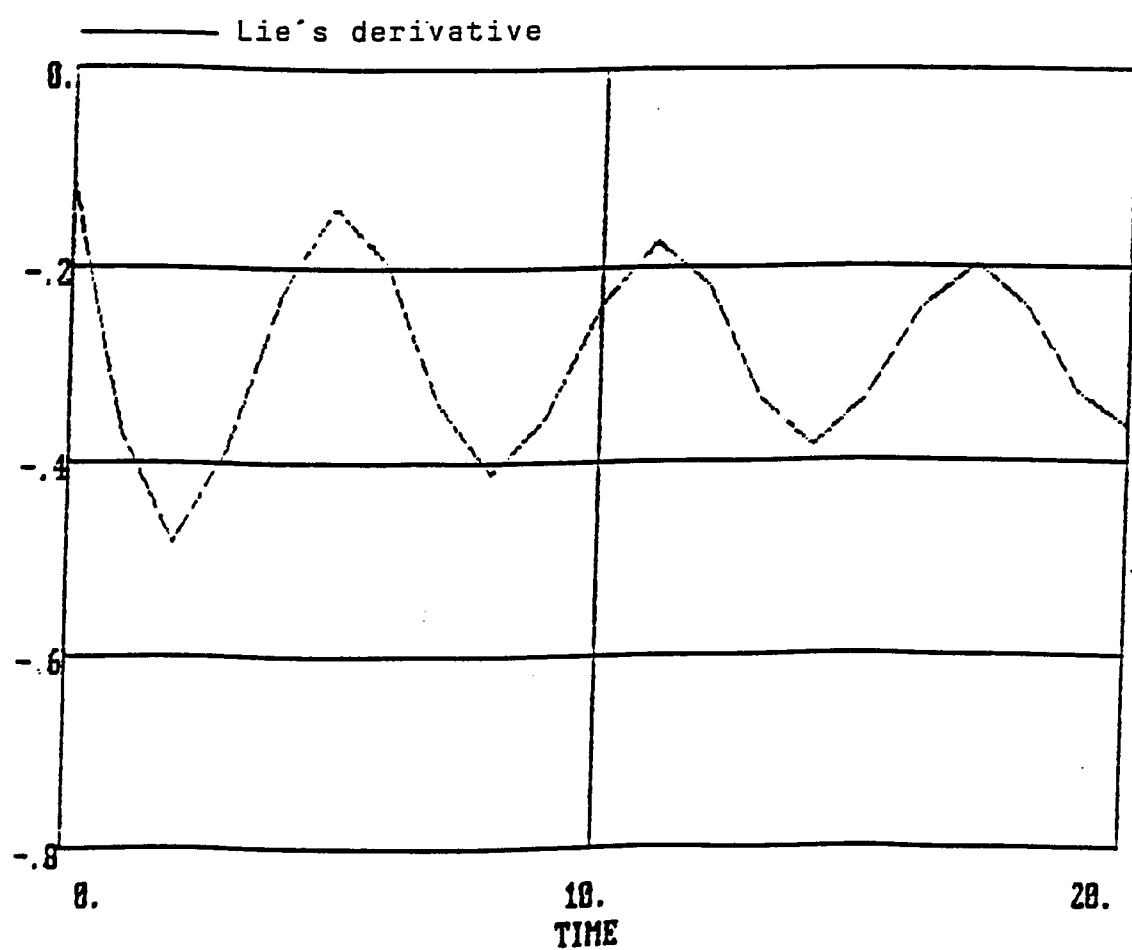
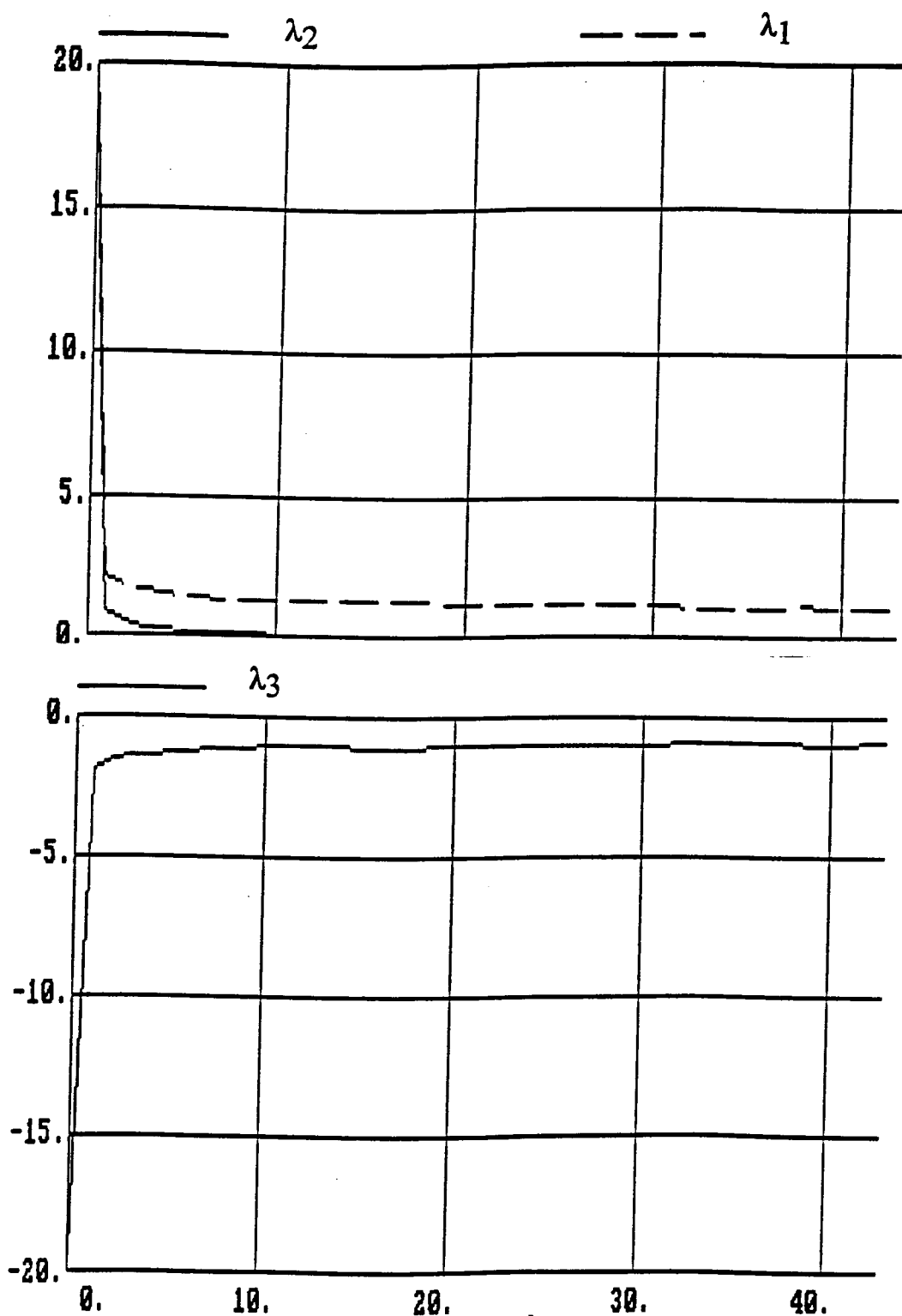


Figure 4. Lie's derivative

Table 1. The influence of the value of the control parameter (β) on the equilibrium properties

β	EIGENVALUES		Derivative	Equilibrium (x^* , β)
	real λ	conjugate pair ($\bar{a} \pm bi$)	$\delta \bar{a} / \delta \beta$	
0.2	-0.78	-0.105 + 1.06 i	0.28	locally and asymptotically stable
0.3	-0.78	-0.81 + 1.03 i	0.27	locally and asymptotically stable
0.6393	-0.077	0. + 0.97 i	0.26	nonhyperbolic, nonstable ¹⁾
0.73	-0.077	0.02 + 0.95 i	0.25	the saddle
0.8	-0.077	0.04 + 0.93 i	0.02	the saddle
0.82	0.01	-0.081 + 0.92 i	0.02	the saddle-focus

1) There is the Hopf bifurcation: the existence of closed orbits in a neighborhood of (x^* , β_0) with the period of approximately $2\pi/b$.



Chaos (+,0,-)

Figure 5. An example of the Lyapunov exponents:

$$\lambda_1 \cong 1.03, \lambda_2 \cong 0., \lambda_3 \cong -0.83$$

The constellation of coefficients: $\beta > \mu_2$

$$\mu_1 = 0.12, \mu_2 = 0.639333, \nu_1 = 0.12, \nu_2 = 1.5, \rho = 1.8, \beta = 0.669333, \\ \gamma = 1, n = 0.02.$$

Table 2. The examples of the long waves

μ_1	μ_2	Steady state growth rate ($\hat{a}_2 = \hat{w}_2$, $\hat{a}_e = \hat{w}_e$)	Relative speed of convergence
0.02	0.5	0.04	low
0	1.	0.04	moderate
-0.012	1.3	0.04	high

Notes: 1. The length of the fluctuations is about 60 years.

2. If $\mu_1 \neq 0$ and $\mu_2 \neq 1$ then the initial vector $(\sigma_0, u_0, v_0) = (\sigma_2, u_2, v_2 - 0.01) = (4.2, 0.75, 0.89)$; if $\mu_1 = 0$ and $\mu_2 = 1$ then the initial vector $(\sigma_0, u_0, v_0) = (\sigma_0, u_e, v_e - 0.01) = (4.2, 0.75, 0.89)$. See (4.8a) above.

3. The magnitudes of the other parameters:

$v_1 = 0.01, v_2 = 0.04, \rho = 0.062, \beta = 0.2, \gamma = 0.02, n = 0.02$.

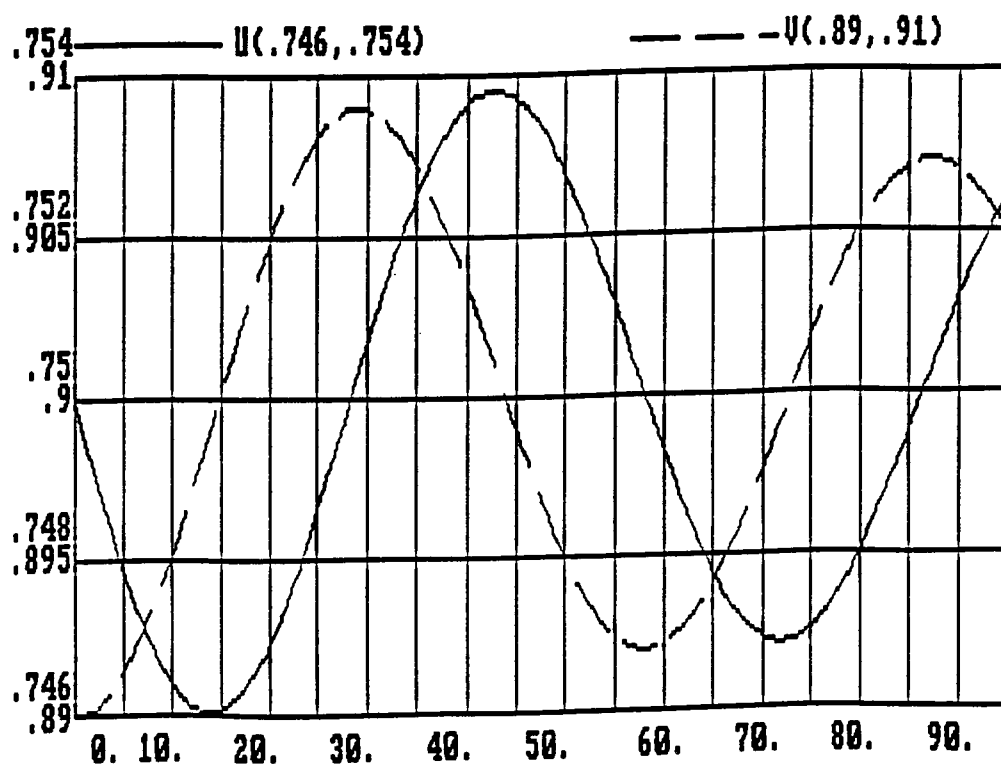


Figure 6. Long wave, $\mu_1 = 0.02, \mu_2 = 0.5$.

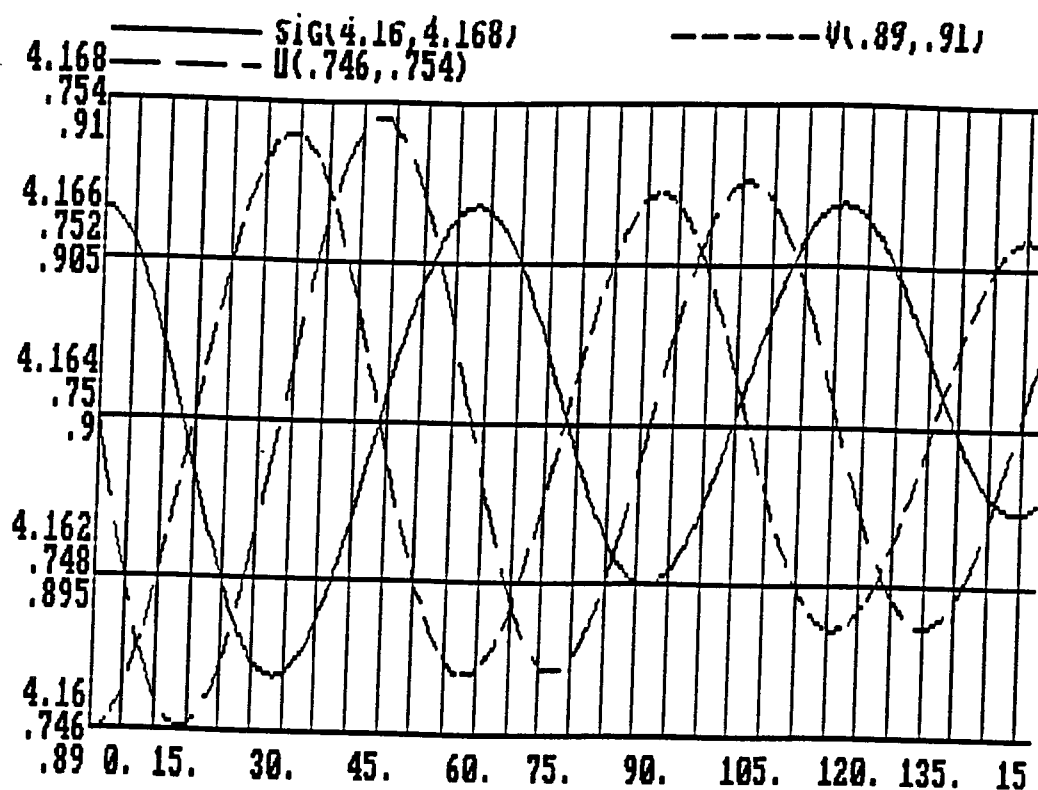


Figure 7. Long wave, $\mu_1 = 0$, $\mu_2 = 1$

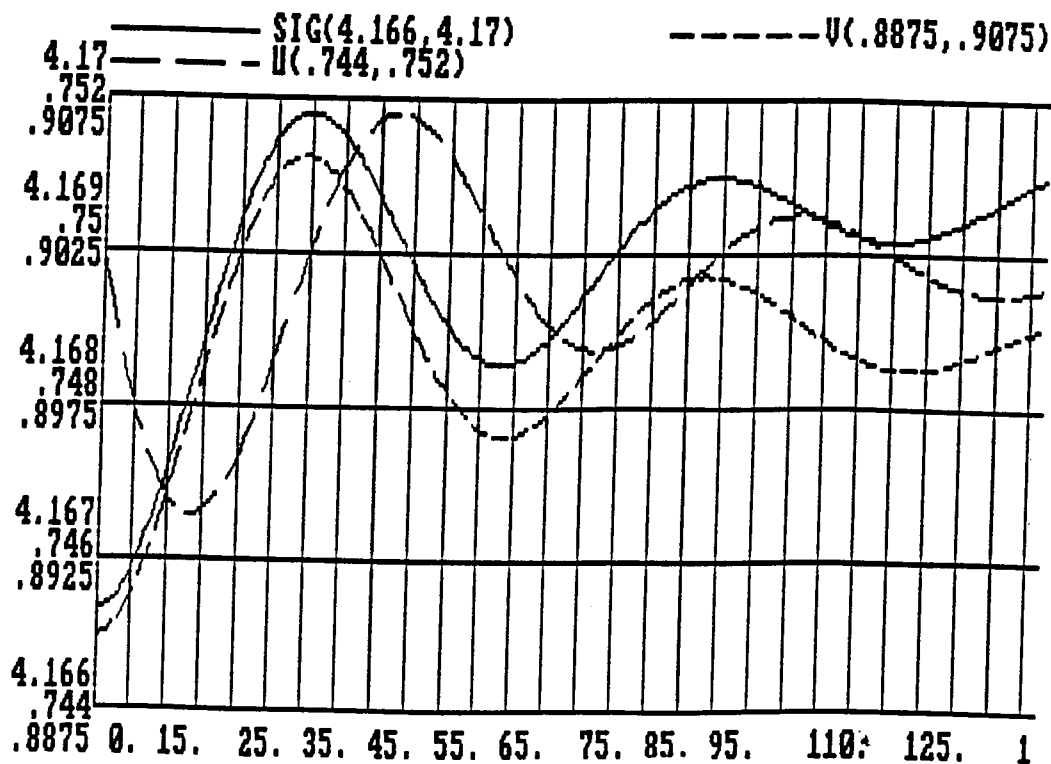


Figure 8. Long wave, $\mu_1 = -0.012$, $\mu_2 = 1.3$

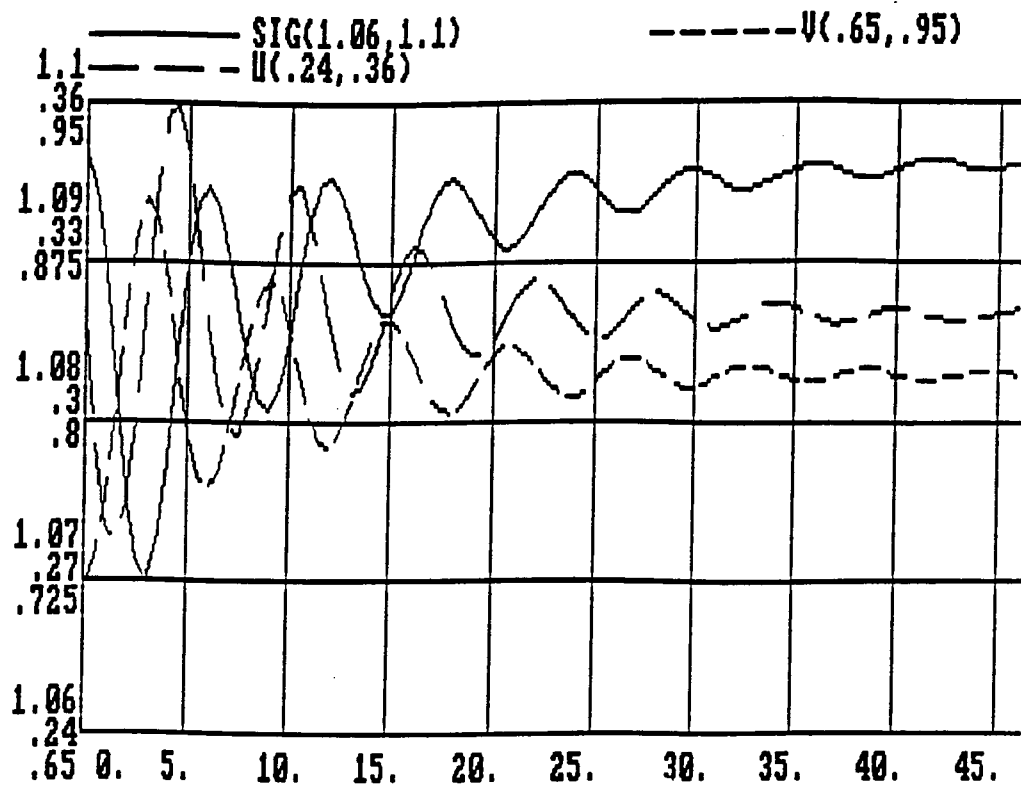


Figure 9. A damping cyclical motion (delays are not introduced)

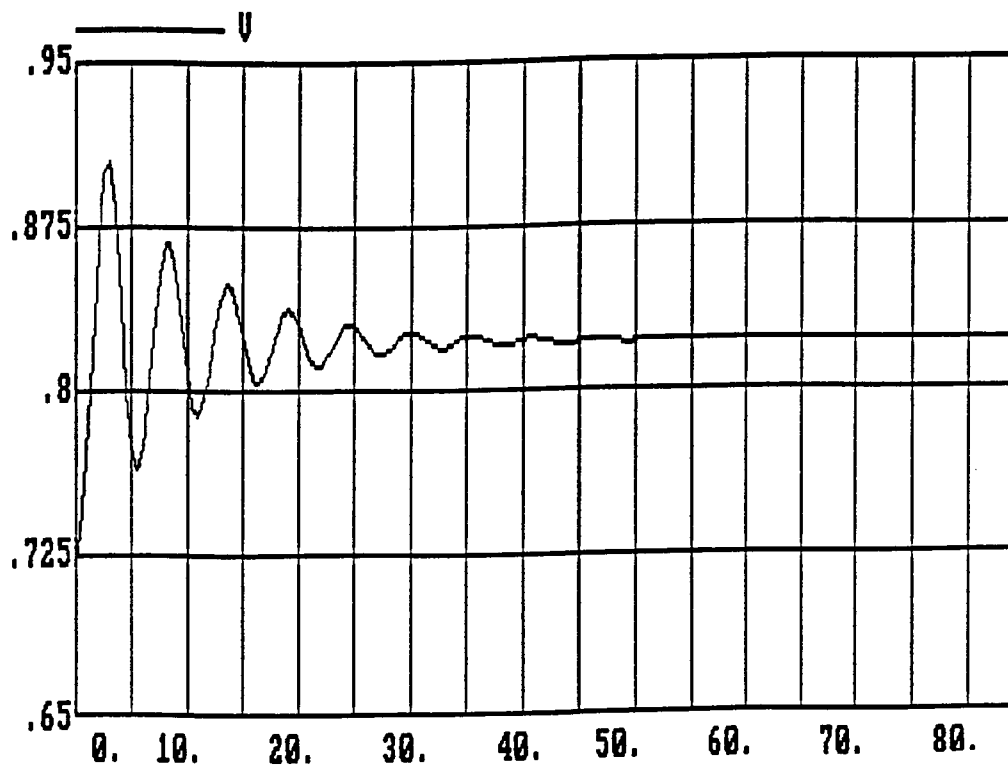


Figure 10. An innovating bargaining ($\hat{a}$ versus $\hat{K}/\hat{L}$) with $DEL = 0.25$

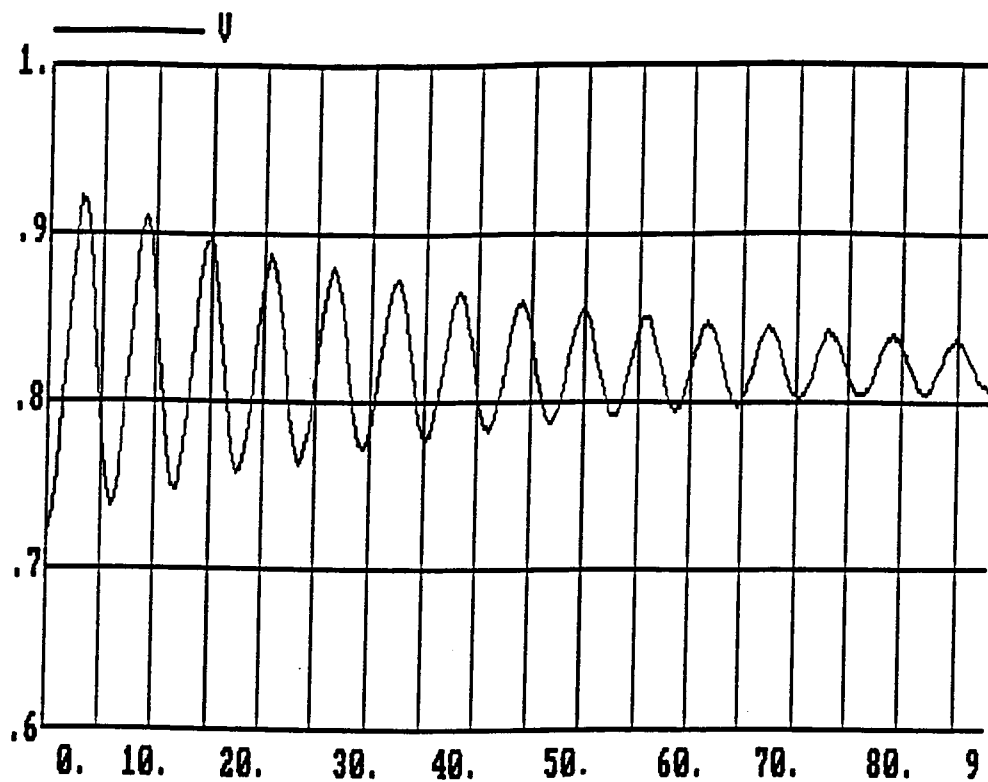


Figure 11. An automating bargaining ($\hat{K}/\hat{L}$ versus u) with $DEL = 0.25$

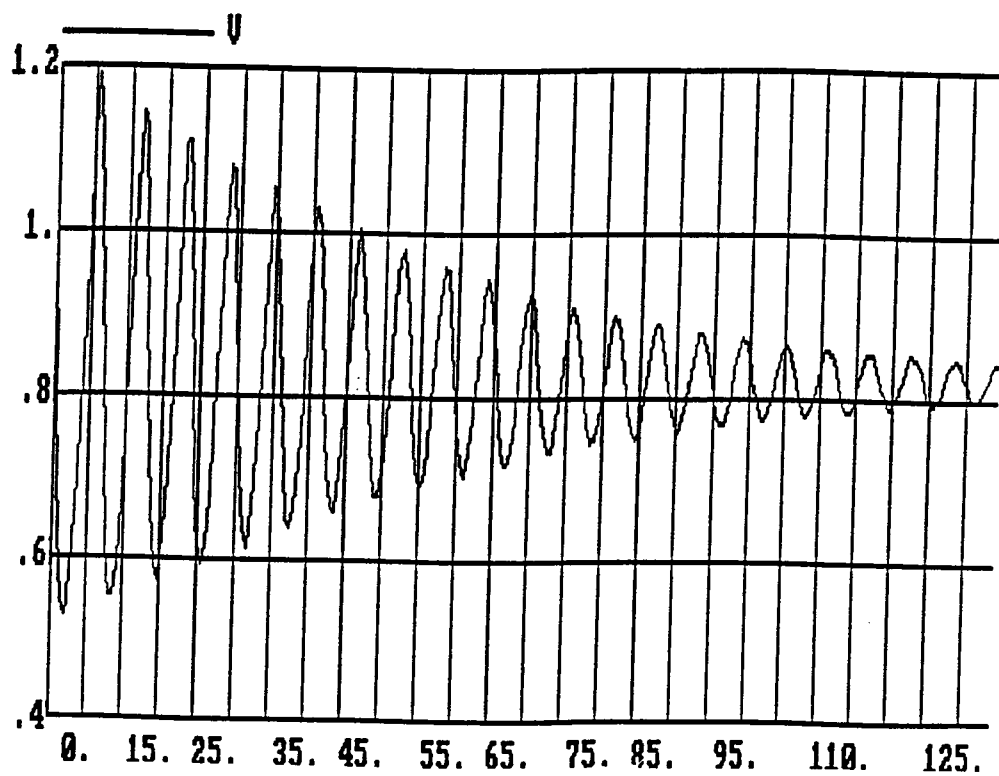


Figure 12. An employment bargaining ($\hat{v}$ versus $\hat{a}$) with $DEL = 0.25$

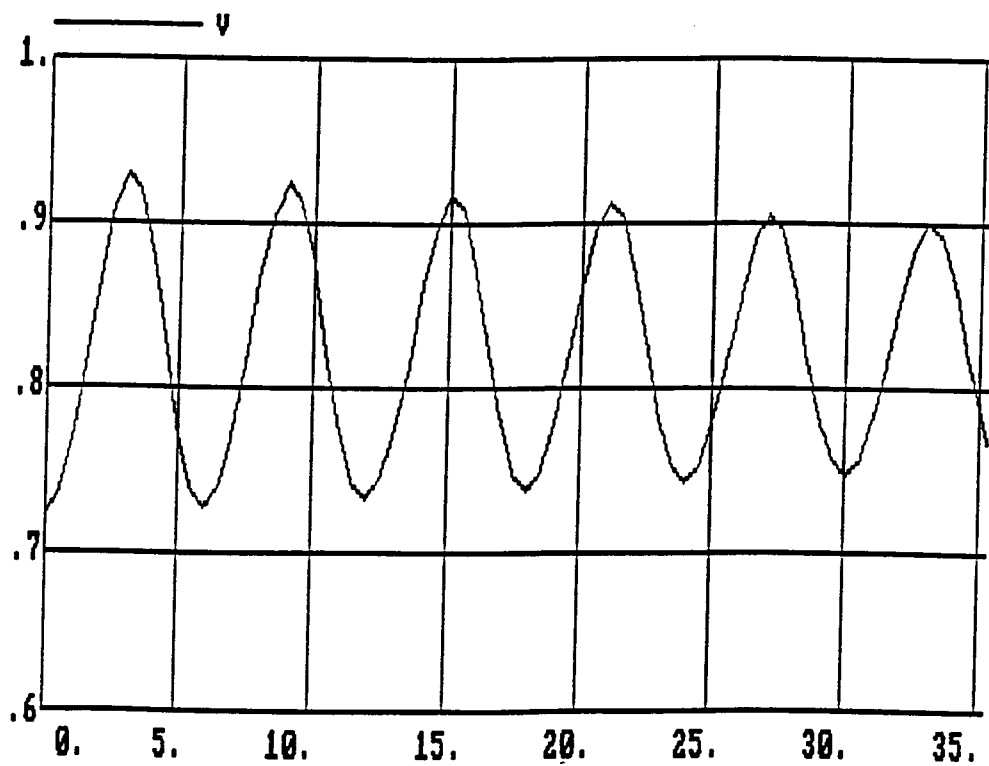
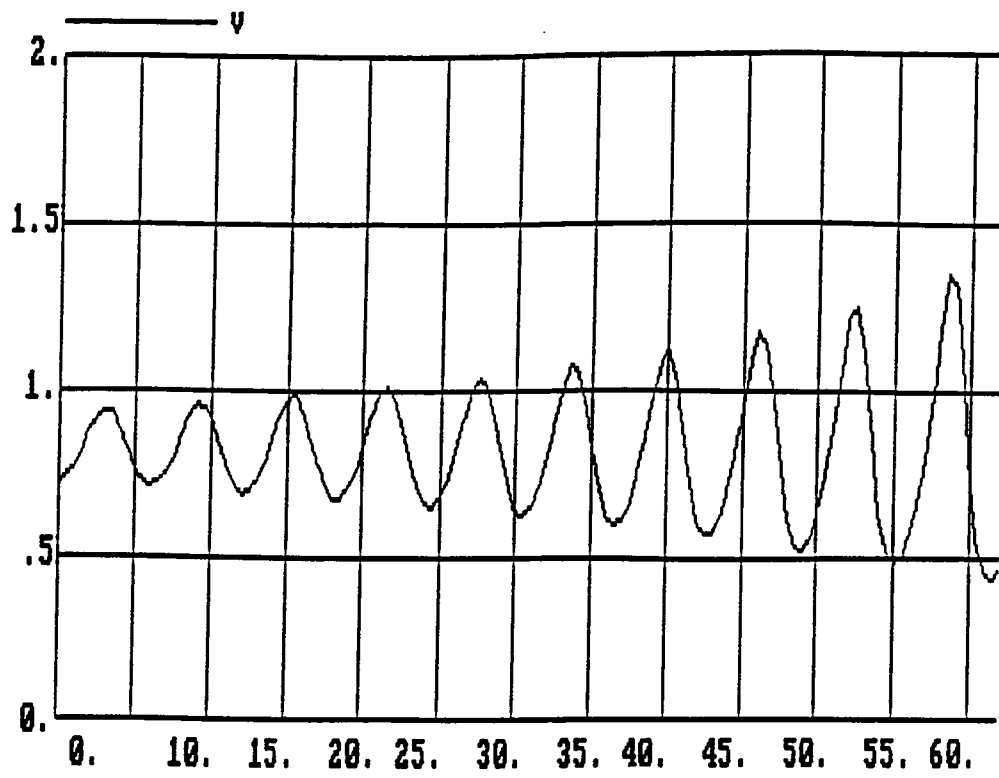


Figure 13. A wage bargaining ($\hat{w}$ versus v) with $DEL = 0.25$ (exploding fluctuations) and with $DEL = 0.18$ (converging fluctuations)

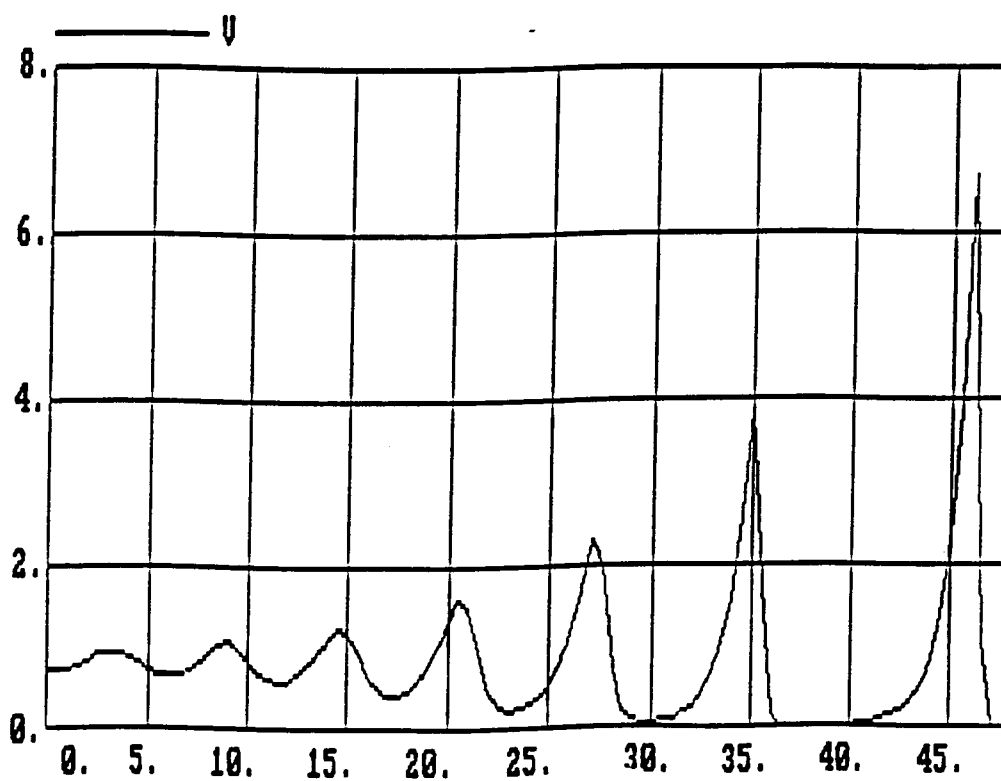
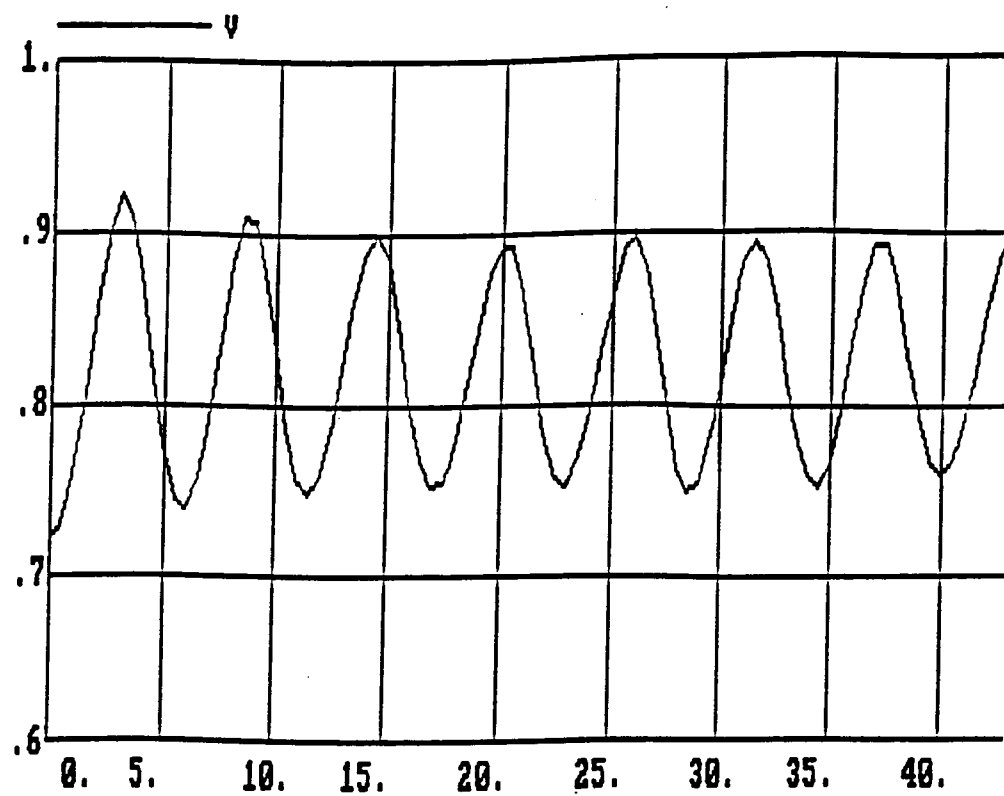


Figure 14. Simultaneous bargaining with $DEL = 0.1$ (converging fluctuations) and with $DEL = 0.25$ (exploding fluctuations)

Table 3. Effects of the bargaining delays

Elementary bargaining	Variable	Factor	Delay (years)	Fluctuation
wage	$\hat{w}$	v	0.25 0.18	exploding converging
automating	$\hat{K/L}$	u	0.25	converging
innovating	$\hat{a}$	$\hat{K/L}$	0.25	converging
employment	$\hat{v}$	$\hat{a}$	0.25	conveging
Simultaneous bargaining			0.25 0.10	exploding converging

Notes: 1.The equilibrium: $(\sigma_2, u_2, v_2) = (1.0968, 0.32, 0.822)$.

2.The initial vector $(\sigma_2, u_2, v_2 - 0.1)$.

3. The magnitudes of the parameters:

$\mu_1 = 0.12, \mu_2 = 0.8, v_1 = 0.12, v_2 = 1.5, \rho = 1.8, \beta = 0.2, \gamma = 1, n = 0.02$.

Table 4 Ranks of technologies according to their profitability (the first game)

Tech-nique	Year									Sum	Final rank
	0	1	2	3	4	5	6	7	8		
1	3	4.5	4	4	4	4	4	4	2	33.5	3
2	5	6	5	5	5	5	5	5	4	45	5
3	8	7	6	6	6	6	6	6	6	57	7
4	9	9	9	9	9	9	9	9	9	81	9
5	6	8	8	8	8	8	8	8	8	70	8
6	1	2	7	7	7	7	7	7	7	52	6
7	2	1	1	1	1	1	1	1	1	10	1
8	7	4.5	3	3	3	3	3	3	5	34.5	4
9	4	3	2	2	2	2	2	2	3	22	2

Table 5 Ranks of technologies according to the total labour intensity (the first game)

Tech-nique	Year									
	0	1	2	3	4	5	6	7	8	9
1	3	3	3							
2	2	2	2	2	2	2	2	2	2	2
3	1	1	1	1	1	1	1	1	1	1
4	2	2	2	2	1	1	1	1	1	1
5	1	1	1	1						
6	3	3	3	3						
7	3	3	3	3	3	3	3	3	3	3
8	1	1	1.5	2	2	2	2	2	2	2
9	2	2	1.5	1	1	1	1	1	1	1

Table 6 Ranks of technologies according to their profitability (the second game)

Tech-nique	Year											Sum	Final rank
	0	1	2	3	4	5	6	7	8	9	10		
1	3	4	3	2.5	5	4	1	2	4	4	2	34.5	3
2	5	6	5.5	6	6	5	2	4	5	5	3	52.5	4
3	8	7	7	7	7	6.5	4	5	6	7	7	63.5	6
4	9	9	9	9	9	9	9	6	3	3	9	84	9
5	6	8	8	8	8	8	8	3	2	2	8	69	8
6	1	1	2	2.5	4	6.5	3	1	1	1	6	29	1
7	2	2	1	1	1	1	5	7	7	6	1	34	2
8	7	5	5.5	5	3	3	7	9	9	9	5	67.5	7
9	9	3	4	4	2	2	6	8	8	8	4	58	5

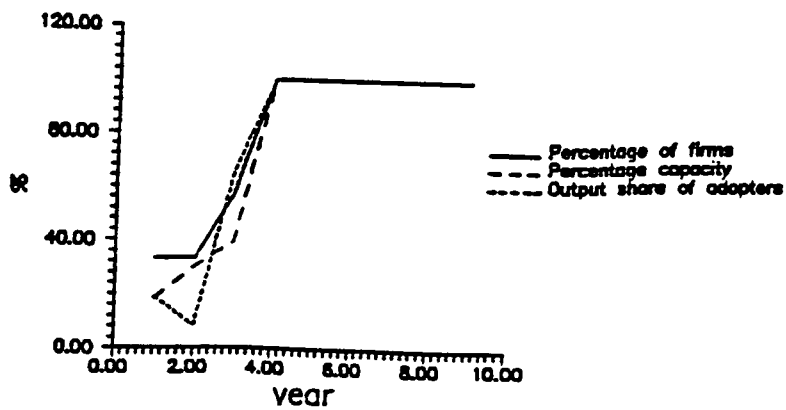


Figure 15
Diffusion curves (game 1)

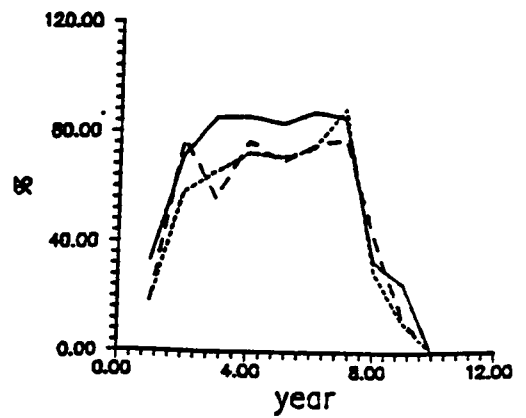


Figure 16
Diffusion curves (game 2)

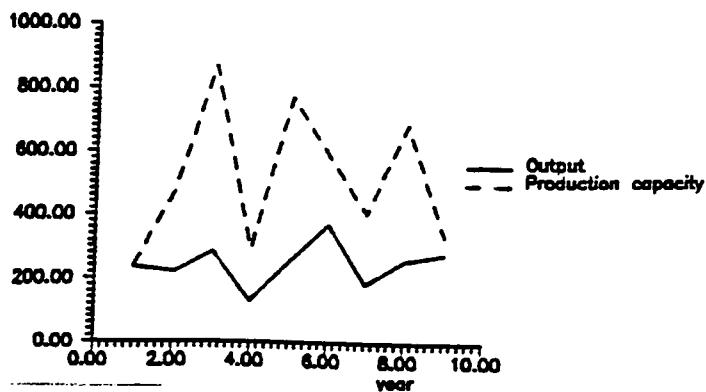


Figure 17
Output and production capacity
in physical terms (game 1)

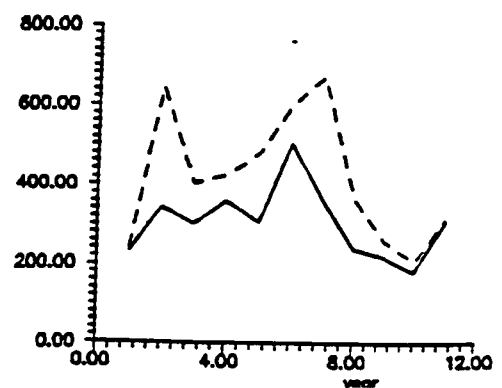


Figure 18
Output and production capacity
in physical terms (game 2)

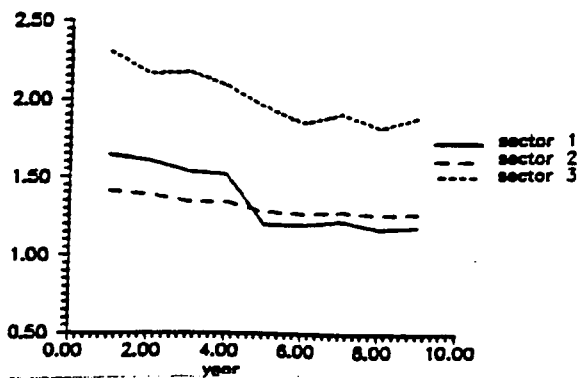


Figure 19
Total labour input coefficients by
sectors (man-year/cost units) (game 1)

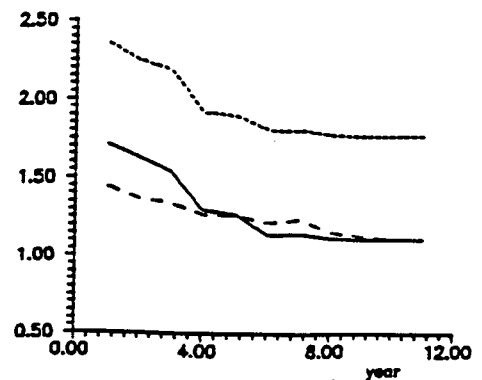


Figure 20
Total labour input coefficients by
sectors (man-year/cost units) (game 2)

Table 7 Ranks of technologies according to the total labour intensity (the second game)

Technique	Year											
	0	1	2	3	4	5	6	7	8	9	10	11
1	3	3	3	3	2	2						
2	2	2	2	2								
3	1	1	1	1	1	1	1	1	1	1	1	1
4	2	2	2	2	2	2	2	2	2	2		
5	1	1	1	1	1	1	1	1	1	1	1	1
6	3	3	3									
7	3	3	3	3	3							
8	1	2	1	2	2	2	2	2	2	2	2	2
9	2	1	2	1	1	1	1	1	1	1	1	1

Table 8 The relationships between net price (v_j^t), labour intensity (l_j^t) and gross output (q_j^t) in the first game (over years 1-9) and in the second game (over years 1-11). Ranks in parentheses.

	Index for game 1			Index for game 2		
	v_j^9/v_j^1	l_j^9/l_j^1	q_j^9/q_j^1	v_j^{11}/v_j^1	l_j^{11}/l_j^1	q_j^{11}/q_j^1
Sector 1	.8(1)	.6(1)	1.4(3)	.6(1)	.5(1)	1.8(3)
Sector 2	1.9(3)	1.6(3)	1.2(2)	3.2(3)	.9(3)	1.3(1)
Sector 3	1.0(2)	.9(2)	.7(1)	.7(2)	.89(2)	1.4(2)