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A Pissarides-Dixit-Stiglitz model

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Abstract

We integrate two workhorse models in economics: The monopolistic competition model of Dixit and Stiglitz and the search unemployment model of Pissarides. Many results of the original models survive. New results concerning the effects of changes in labour (goods) market parameters on goods markets (the labour market) variables are obtained and compared to the literature.

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1. Introduction

In his theory of search unemployment Pissarides (1990) links the essentials of his theory to the neo-classical production function and the neo-classical growth model. In this paper we link Pissarides' theory to the monopolistic competition model of Dixit and Stiglitz (1977).

Many economists believe that fixed costs and product differentiation are very attractive features of the latter model, which have made it a useful tool in new trade, growth theory, macroeconomics and regional economics. In particular, in economic policy debates the number and size of firms is often related to the unemployment issue. However, the number and size of firms are not determined in models using the neo-classical production function and search unemployment does not appear in the Dixit-Stiglitz model. As many economists argue the search theory of unemployment has received much empirical support. It seems to be a worthwhile effort to link Pissarides' theory to the monopolistic competition model, thus joining two of the workhorse models in economic theory.

TABLE 1 ABOUT HERE

Table 1 summarizes literature on imperfect competition and *non-monetary* theories of unemployment, which comes closest to our contribution. The first column lists models of the Dixit-Stiglitz type. Unemployment in this column is based on pessimistic expectations in Weitzman (1985) and Dutt and Sen (1997). In Weinrich (1993), using the Blachard and Kiyotaki (1987) variant of the Dixit-Stiglitz model, unemployment is based on the use of an effort function that is continuous in wages and unemployment. In Matusz (1996, 1998) it is based on the Shapiro-Stiglitz model of efficiency wages. In deGroot and van Schaik (1998, 2000) dual labour markets with unemployment arise because of an efficiency wage relation. In de Groot (2001) dual labour markets arise because of union bargaining. The last model mentioned in the first column of the table is the focus of this paper.

In the second column, models with other forms of monopolistic competition than that of Dixit-Stiglitz are listed. Weitzman (1982) uses a Hotelling model and unemployment is based on pessimistic expectations of the self-fulfilling prophecy type. Bräuninger (2000) considers unemployment based on bargaining. Bargaining is about profits and rents from being employed. Unlike the Pissarides model there are no hiring costs creating rents for firms but rather – unlike the Dixit-Stiglitz model - profits are positive because the model has a large, exogenous number of firms and no fixed costs. There have been other attempts to relate *search unemployment* and monopolistic competition in the literature. One group of attempts is related to the Aghion and Howitt (1994, 1998 chap.4) quality ladders approach. In that approach the number of firms is given because there is vertical product differentiation. We use the Dixit-Stiglitz approach of horizontal differentiation because it has an endogenous number of firms. Bean and Pissarides (1993) adjust the Pissarides (1990) model to an overlapping-generations growth model by fixing the employment duration to last one period. Bean and Pissarides then go to analyse the effect of savings shocks on employment and growth; this topic is not considered here. In their model firms play Cournot in each differentiated product. Table 1 indicates which gap in the literature this paper fills.

In this paper, we investigate to what degree the comparative *static* results of Pissarides (1990) and Dixit-Stiglitz (1977) are unchanged and which results on the

number of firms can be added. We limit our comparison with the literature to that of other *static* models. Results from literature that uses either a different theory of unemployment or a different model of monopolistic competition can be compared with our results more straightforwardly.

Matusz (1998) has linked the Dixit-Stiglitz model to efficiency-wage unemployment of the Shapiro-Stiglitz type. In the non-shirking constraint the number of varieties appears. Therefore country size or international trade increasing the market size yielding more variety relaxes the non-shirking constraint. By implication the unemployment needed to deter shirking is lower. In Matusz (1996) an increase in market size increases productivity because of an increased number of intermediate products, which allows for higher wages and less equilibrium unemployment again via a non-shirking constraint. In this paper we find no market size effects of international trade on the unemployment percentage rate for several reasons. First, with differentiated consumer goods in our model the productivity effect of Matusz (1996) is absent in our set up. Second, having no non-shirking constraint in the Pissarides part of our model, variety cannot have such a prominent role in relation to unemployment as it has in Matusz (1998). Other channels, from market size to the rate of unemployment, - discussed verbally by Matusz (1996)[i] - are also not modelled here because there is hardly any empirical indication that they are relevant.

Unlike Dutt and Sen (1997) we do not find that increased bargaining power of unions increases demand and therefore employment. The reason is that our model, unlike the Weitzman (1985) type of model used by Dutt and Sen, is not demand constrained and allows for real-wage flexibility and entry.

Weitzman (1982) has emphasised the negative relation between (expected) unemployment and real wages stemming from a move up the average cost curve when unemployment gets higher. Our model also allows for this relation, but also for the opposite one depending on what type of exogenous shocks cause the move. If the shock comes from interest or separation rates there is a negative relation. If, however, the shock comes from unemployment benefits and bargaining power then there is a positive relationship.

Vogt (1996) has criticised monopolistic competition models because of their property to translate wage increases completely into price increases (see Weitzman (1985), Blanchard and Kiyotaki (1987), Matusz (1996, 1998)). He circumvents this point by developing a model of potential competition. A different way of avoiding this is to introduce theories of unemployment into those of monopolistic competition. In this paper, the problem is avoided by introducing search unemployment into a model of monopolistic competition. Wage increases can either increase prices or decrease hiring costs as in Pissarides (1990).

Our model has higher employment under monopolistic than under perfect competition as in Blanchard and Kiyotaki (1987) where a real balance effect with endogenous labour supply causes this result.

Other results are presented as propositions below and summarised in section 5. Which of the results are better – ours, or those in the literature - is an empirical question that goes beyond the scope of this paper.

2. The model[ii]

Trade in the labour market

From the Pissarides (1990) model we use the matching function $m\,L = m(uL, vL)$, where L is the total number of employed and unemployed workers, u is the unemployment rate, v is the rate of vacancies and mL is the number of matches produced by this function. The function is assumed to be increasing in both arguments, concave and linearly homogenous.[iii] Defining labour market tightness as $\theta \equiv v/u$, division of the matching function by vL yields $q(\theta)= m(u/v,1)$ as the probability of a firm to find a worker for a vacancy and $\theta q(\theta) = m/u = m(1, v/u)$ as the probability of an unemployed worker to find a job. A shock is a percentage rate s at which $(1-u)L$ employed workers loose their job by assumption in every period. Therefore $s(1-u)L$ workers go from a job into unemployment every period. On the other hand $\theta q(\theta)uL$ unemployed workers expect to find a job each period. A labour market steady state equilibrium is defined as a situation where the numbers of workers going into and out of unemployment are equal and expectations turn out to be true, i.e. $s(1-u)L= \theta q(\theta)uL$. Solving this equation for u yields the Beveridge or UV curve (lower indices referring to variables indicate partial derivatives):

$$u = \frac{s}{s+\theta q(\theta)}, u_s > 0, u_\theta < 0 \qquad (1)$$

Multiplying equation (1) by θ yields an equation for the vacancy rate because $u\theta=uv/u=v$:

$$v = \frac{s}{s/\theta + q}, v_s > 0, v_\theta > 0 \qquad (1')$$

Equation (1) and (1') are drawn in the lower right quadrant of figure1.

Government or unemployment insurance

The government is assumed to pay unemployment benefits z to each unemployed worker. The financing of this is not explicitly treated in Pissarides (1990). We show how this can be modelled to keep Pissarides' results intact. Total expenditures of the government or unemployment insurance for unemployment benefits are zuL. It will turn out that the incentives are ultimately unchanged if both the employed and the unemployed pay a tax or insurance premium t to finance the unemployment benefits. Revenue then is tL. From the balanced budget assumption we make it follows that $tL = zuL$ and therefore $t = zu$. Workers therefore receive $w-t = w – zu$ and unemployed benefits are $z - t = z – zu$. As z is considered to be a policy variable, the budget equation determines the value of t, whereas u is determined in the general equilibrium part of the model below.

Households and workers

The present value, discounted at rate r, of the expected income stream of an unemployed and an employed worker, U and E respectively, are: $U = [z - zu + \theta q(\theta)(E-U)]/r$ and $E=[w + s(U-E)]/r$. $E-U$ is the income difference an unemployed worker can gain by finding a job with probability $\theta q(\theta)$. $U-E$ is the corresponding loss by a worker from losing his job with probability s. These two equations can be solved for E and U explicitly:

$$U = \frac{(r+s)z + \theta q(\theta)w}{r+s+\theta q(\theta)} / r - zu/r \ , \ E = \frac{sz + [r+\theta q(\theta)]w}{r+s+\theta q(\theta)} / r - zu/r$$

Households are assumed to have love-of-variety preferences $y = \left[\int_{i=0}^{n} c_i^{\alpha} di\right]^{\frac{1}{\alpha}}$, with $0 < \alpha < 1$, on a continuum of goods ranging from zero to n, the (integral measure of) the number of firms.[iv] It is well known that this specification of preferences leads to a constant elasticity of the inverse demand function, α-1, and to relative demand of goods independent of the income earned by employed or unemployed persons.

Firms

The present-discounted value of a vacancy is $V = [-\gamma + q(\theta)(J-V)]/r$. It consists of the hiring costs γ and the net return of transferring the vacancy V into a job with value J expected with probability $q(\theta)$. r is the discount rate. As the value of the vacancy is zero in equilibrium, we get $J = \gamma / q(\theta)$: the value of a job is equal to the vacant job costs γ multiplied by the expected duration of the vacancy. When considering the firms' hiring costs we must consider that the occupied job may be separated from the worker again with probability s. The current value of the expected value of a job therefore is $(r + s)J = (r + s)\gamma/q(\theta)$. These are labour costs that are added to the real wage received by the worker. Labour costs then equal $w + (r + s)\gamma/q(\theta)$. Pissarides (1990) links the above to the neo-classical production function.[v] Here we link it to the model by Dixit and Stiglitz (1977). If the temporary utility function is discounted and integrated we may get the inter-temporal utility function $\int_{t=0}^{\infty} e^{-rt} y(t) dt$. It is well known from endogenous growth theory or the theory of optimal growth that, in the absence of productivity growth, the value of consumption will be stationary and the interest rate will equal the discount rate r. This seems to be the shortest way to determine the interest rate.

Technologies are defined by $l_i = f + a\, x_i$, with $a, f > 0$. The left side represents demand for labour to produce good i, f is the fixed part and ax_i is the variable part of labour demand. As all goods are assumed to be identical in the utility function and in the production technology, their prices and quantities will be the same.[vi]

Total labour demand is $nl_i = n(f + a\, x_i)$. Equating this to the employment $(1-u)L$ yields $(1-u)L = nl_i = n(f + a\, x_i)$. Solving the latter equation we find the rate of unemployment linked to the number of firms as:

$$u = \left(L - nl_i\right)/L = [L - n(f + ax_i)]/L \qquad (2)$$

Proposition 1*: There is a partial negative relation between the rate of unemployment and the number of firms: The larger the number of firms, the lower the unemployment rate (ceteris paribus), or the lower the unemployment rate the more firms can be in the market.*

Profits of a firm, which are zero in equilibrium, are defined (dropping index i) in nominal terms as

$$p(x)x - [W + p(r + s)\gamma/q(\theta)](f + a\, x) = 0 \qquad (3)$$

W is the nominal wage rate and real hiring costs are made nominal by multiplying their real value with the price. With the exception of Vogt (1996) all the other *static* monopolistic competition models mentioned above do not take hiring costs into account. The first-order condition of profit maximisation is:

$$p\alpha = a[W + p(r+s)\gamma / q(\theta)] \qquad (4)$$

In the model by Pissarides (1990) the neo-classical way to get full employment with decreasing real wage costs is blocked by search frictions. In monopolistic competition models without hiring costs this road is absent anyway because real wages are fixed as in (4) (see Weitzman (1985), Blanchard and Kiyotaki (1987), Matusz (1996, 1998). The Dixit-Stiglitz road to full employment used to be entry whenever profits are positive, according to equation (2) with $u=0$. In Weitzman (1982) $u>0$ comes in exogenously via the assumption that firms expect aggregate demand to be (in symbols of our model) $u^e wL$, where all the three variables are given exogenously. With this expected demand, a self-fulfilling prophecy yields the unemployment rate $u=u^e>0$. In our model, however, the Dixit-Stiglitz road to full-employment is present but limited by Pissarides' search friction, resulting in an endogenous value of unemployment $u > 0$ incorporated in equation (2).

Wages

There are two sorts of rents in Pissarides' model: on occupied jobs, indexed *j*, employed workers do not have to search and therefore have an income rent of $E_j - U$ and firms do not have to incur hiring costs and therefore have a rent J_j-V. Bargaining these rents is assumed to determine real wages. This is done by choosing the real wage by maximising the function $(E_j-U)^{\beta}(J_j-V)^{1-\beta}$ with β as the bargaining power of workers and $1-\beta$ that of firms, $V=0$, $E_j=[w_j - zu + sU]/(r+s)$, U according to the explicit solution given above, and $J_j= \{[p(x)x]/ (f + a x)] -W_j\}/[(r + s)p]$ which equals $\gamma/q(\theta)$ when setting profits in (3) equal to zero.[vii] E, U and V are as in Pissarides (1990). The value for J differs from Pissarides' model because we have replaced the neoclassical production function by elements of the Dixit-Stiglitz model. The result of the maximisation with respect to the real wage in its general form is identical to that of Pissarides in that workers get a share β of the sum of the rents to be distributed: $E_j-U = \beta(E_j-U + J_j-V)$. Insertion of the values for E_j, U, J_j and V yields the solution for real wages:[viii]

$$w_j = (1-\beta)(rU + zu) + \beta \frac{x}{f + ax}$$

Insertion of $E_j-U = \beta(J_j-V)/(1-\beta)$ from the general form of the bargaining result and $J=\gamma/q(\theta)$ into $rU = [z - zu + \theta q(\theta)(E-U)]$ yields $rU = z - zu + \theta\beta\gamma/(1-\beta)$. Insertion of rU into the above wage result yields[ix]

$$w_j = (1-\beta)z + \beta\left(\frac{x}{f + ax} + \theta\gamma \right) \qquad (5)$$

The last term indicates that workers participate in the hiring costs saved on occupied jobs compared to vacancies. The second but last term is net revenue or output per worker as in Pissarides (1990) - where the output-per-worker term is $f(k) - (r+\delta)k$ -, but here without capital cost as in the Dixit-Stiglitz model. The unemployment tax, zu, has dropped out only in the very last step of the calculation yielding (5). This shows that the Pissarides approach is consistent with an explicit financing scheme for the unemployment benefit if both unemployed and employed workers have the same reduction of their gross payments w and z respectively. Then the difference of going from a status of unemployed to employed workers is unchanged and all incentives are exactly as in Pissarides' model.

This model is kept as simple as the basic workhorse models were. We resist the temptation to endogenize the bargaining power parameter, or distinguish between the love-of-variety and the price elasticity parameters, or endogenize the mark-up. These extensions can be taken on board when applications require to do so.

3. The Equilibrium Solution: Existence and uniqueness

Equations (1)-(5) determine the five variables of the model when goods produced serve as numéraire *(p=1): u, n, x, θ and w*. Insertion of wage plus hiring cost per worker from (4) into (3) allows to solve for the equilibrium quantity:

$$x = \frac{f\alpha}{a(1-\alpha)} \qquad (6)$$

Proposition 2: *The equilibrium quantity of the model is independent of the labour market variables stemming from Pissarides' part of the model (unemployment benefit z, hiring costs γ, unemployment rate u, vacancies v, separation rate s, power parameter β).*

Clearly, this result is due to the fact that the firm part of the Dixit-Stiglitz model is merely changed by adding hiring costs to the wage rate: this term drops out when solving (3) and (4) for the quantity. Next, we replace real revenue or output per worker in (5) by the labour cost term from (3) and solve for w:

$$w = z + \frac{\beta}{1-\beta}\frac{r+s+\theta q(\theta)}{q(\theta)}\gamma \qquad (5')$$

This equation essentially has the real wage as a function of the $v/u=\theta$ ratio. This equation is drawn as the BB curve in the upper right quadrant of Figure 1.[x] To solve the system the next steps serve to get a second equation of this type. Solving (2) for n using (6) to replace x, we get:

$$n = \left[L - \frac{sL}{s+\theta q(\theta)}\right]\frac{1-\alpha}{f} = L(1-u)\frac{1-\alpha}{f} \qquad (2')$$

This is a function $n(\theta)$ or $n(u)$. The $n(u)$ function is drawn in the lower left quadrant of Figure 1. Dividing (4) by the price and solving for the real wage yields:

$$w = \frac{\alpha}{a} - \frac{r+s}{q(\theta)}\gamma \qquad (4')$$

Larger hiring costs *(r+s)J* imply lower wages according to (4') as in Pissarides' model when interest is given. Here the model resembles Pissarides' because the zero-profit condition in his model – rewritten in footnote 5 above - implies constant labour costs as long as $r = f'(k)-\delta$ and therefore *k* are constant. By implication wages *w* always move in the opposite direction of hiring costs, $(r+s)\gamma/q(\theta)$, in Pissarides' model and in ours. This clearly differs from monopolistic competition models without hiring costs where wage increases lead to higher prices under constant returns to scale (see Weitzman (1985), Blanchard and Kiyotaki 1987, equation (12'), Matusz (1996, 1998)). Vogt (1996) argues that this property of monopolistic competition models is undesirable for empirical reasons. He develops a potential competition model in which sunk costs deter entry of a potential Bertrand competitor in homogenous goods resulting in a demand curve with a kink at a limit price. This ensures that prices, quantity and employment are independent of cost increases within certain limits. Our combination of the models by Pissarides and Dixit-Stiglitz generates another, though less strict, separation of wages from prices because costs, price and employment are still interdependent. It is only that increasing *real* wages decrease hiring costs rather than increasing prices. Equation (4') is drawn as a function $w(\theta)$ in the upper right quadrant of figure 1, indicated as the MM curve. It is also drawn in the upper left quadrant of figure 1 with wages as a function of hiring costs.

INSERT FIGURE 1 OVER HERE

The intersection of lines BB and MM determines the wage and the tightness rate in the upper right quadrant, and hiring costs in the upper left quadrant. Given the rate of tightness thus determined, the solution for the rates of unemployment and vacancies can be found in the lower right quadrant. The rate of unemployment then determines the number of firms in the lower left quadrant.

Equations (5') and (4') are two functions $w(\theta)$. Replacing labour costs in (4) using (5') or equating (4') and (5') we get:

$$\alpha = a\left[z + \frac{1}{1-\beta}\frac{r+s}{q(\theta)}\gamma + \frac{\beta\gamma\theta}{1-\beta}\right] \qquad (7)$$

The left side is marginal revenue and the right side is marginal cost. In figure 2 both functions are drawn. The left side is denoted as MR and the right side as MC in figure 2. MC is increasing in θ and may have a negative second derivative in θ.[xi]

INSERT FIGURE 2 OVER HERE

The MC curve starts at *az* if $\theta=0$ and $\lim_{\theta\to 0} q = \lim m(1/\theta, 1) = \lim m(\infty,1)=\infty$; otherwise it starts above *az*. As θ goes to infinity the MC curve also goes to infinity. Thus, the MC curve either intersects once or not at all. Therefore we have a unique or no equilibrium.

Proposition3*: The existence of a unique equilibrium is guaranteed if* $az < \alpha$*. This implies a positive equilibrium value for the tightness ratio* $v/u=\theta$*. The fixed cost parameter f and the size of the economy, L, have no impact on the value of* $v/u=\theta$*.*

If, however, $az \geq \alpha$, the tightness ratio is zero, there are no vacancies and unemployment is 100% according to equation (2). With no output, *z* cannot be paid. Therefore this cannot be an equilibrium situation.

4. Comparative static analysis

The following changes in the determinants of labour costs drive up the MC curve on the RHS of equation (7), but leave the MR curve and the equilibrium *quantity x unchanged* according to equation (6): $d\beta>0$, $d\gamma>0$, $dr>0$, $dz>0$, $ds>0$. This decreases the value of the tightness ratio in figure 2. The UV equation (1), drawn as $u(\theta)$ in the lower right quadrant of figure 1, shifted to higher *u* in case of $ds>0$, then implies that the rate of unemployment goes up, and equation (2) in the lower left quadrant implies that the number of firms goes down. Note that the exogenous changes leave the equilibrium value of the labour cost unchanged according to (4').

From (4') and (1'), which do not contain β and *z*, we can conclude that increased bargaining power and unemployment benefits lead to higher real wages and less vacancies because of the fall in the tightness ratio. In figure 1 this can be seen as only the BB curve shifts (for *dz*) or rotates (for $d\beta$) upward but all other curves do not change their position.

In the upper right quadrant of figure 1 increasing the rate of interest and the strength of shocks rotates the BB curve upward and the MM curve downward. The latter of the two effects on wages is stronger, because from (5), using (6) for *x*, which both do not contain *r* and *s*, we can conclude that increasing the rate of interest and the strength of shocks decrease wages because of the fall in the tightness ratio according to figure 2 and equation (7).[xii]

The impact of the change in hiring costs, $d\gamma>0$, is to decrease tightness and vacancies, but its effect on wages depends on the change of the term $\gamma\theta$ in (5). Applying the implicit function rule to (7) and dividing by θ and multiplying by γ yields

$$\frac{\partial\theta/\theta}{\partial\gamma/\gamma} = -\frac{(r+s)(\gamma/q)+\beta\theta\gamma}{(r+s)\gamma(-1)q^{-2}q_\theta\theta+\beta\gamma\theta}$$

The numerator and the denominator have identical terms up to $-q_\theta\theta/q$. [xiii] This term is smaller than one[xiv]. By implication the percentage decrease of θ is larger in absolute terms than the increase of γ, leading to a fall in wages according to (5). In terms of figure 1 this means that the upward rotation of BB is weaker than the downward rotation of MM.

Proposition 4: *With increases in worker's bargaining power β, hiring costs γ, the discount rate r and unemployment benefits z and the size of shocks s we confirm Pissarides' results of decreasing tightness and increasing unemployment and falling wage rates. We add the result of a decreasing number of firms and increasing unemployment insurance premium both resulting from lower employment.*

In Weitzman (1982) the rate of unemployment is negatively related to real wages: if the (expected) rate of unemployment goes up real wages decrease, because lower expected demand moves the firm up its average cost curve. This is interpreted as a pro-cyclical movement of real wages. In our model a positive or a negative relation may exist depending on the reason that drives up wages: if the exogenous change in question increases (decreases) tightness the unemployment rate goes down (up). For example, an increase in the rate of interest or the separation rate, which decreases tightness and wages and increases unemployment, yields a negative relation as in Weitzman's model. On the other hand, an increase in unemployment benefit *z* or bargaining power β increases wages, decreases the tightness ratio and increases unemployment. A similar negative relation *w(u)* can be found in Weinrich (1993) based on a linear transformation of a Cobb-Douglas effort function that generates constant effort. On this relation u is determined through the use of an equation similar to our equation (4'), which fixes the real wage in the absence of hiring costs (see Weinrich 1993, p.545). Moving the latter equation along the former would result in a negative relation between *w* and *u*. Specific *combinations* of parameter shifts could also generate a positive relation between w and u in the comparative static manner.

Concerning an increase in worker bargaining power Dutt and Sen (1997) find a different result in a demand-constrained model of monopolistic competition cum unemployment of the Weitzman (1985) type. In their model bargaining power raises wage rates. As workers save less than profit earners, demand is increased and so is employment. The authors show that the result can neither be proven nor rejected if a decreasing marginal product of labour exists.

Changes in fixed costs, $df<0$, which is one of two possible versions of exogenous productivity increases, do not change the tightness ratio and therefore the unemployment and vacancy rates and labour costs are unchanged.[xv] Firm size *x* is decreased and the number of firms is increased as in Dixit-Stiglitz. Aggregate output, *nx*, is unaffected. This can be seen from equations (2') and (6).

A decrease in marginal costs via $da<0$, decreases the slope and intercept of the MC curve in figure 2 and shifts up the MM curve in figure 1. The result is a larger value for the tightness ratio, $v/u=\theta$. This reduces the unemployment rate and increases the rate of vacancies according to equation (1) and (1'). The size and number of firms are both increased according to equation (6) and (2'). Wages are increased according to equation (5').

Proposition 5: *(i) Productivity increases in the fixed cost parameter, $df<0$, when we leave labour market variables unchanged, decrease the size of firms while increasing the number of firms as in the Dixit-Stiglitz model. Aggregate output, however, is unchanged. (ii) Pissarides' productivity results appear in our model and are caused by a decrease in the variable labour demand parameter, $da<0$. Employment, tightness, the number and size of firms increase, and so do wages and aggregate output.*

An increase in love-of-variety, $d\alpha<0$, which means that the elasticity of substitution is getting smaller in absolute terms and competition is decreased, shifts down the MR curve in figure 2 and the MM curve in figure 1. The tightness ratio falls, the unemployment rate goes up and the number of vacancies goes down.[xvi] Wages fall according to (5'). Firm size falls according to (6). The effect on the number of firms is ambiguous because the effect via the size of the firm is increasing the number of firms

whereas the effect via the rate of unemployment is decreasing the number of firms as the horizontal intercept of the curve in the lower left quadrant moves outward.

Proposition 6: *A preference induced decrease in substitution and competition decreases firm size as in the Dixit-Stiglitz model, increases unemployment and decreases the tightness ratio and wages. Unlike the Dixit-Stiglitz model, the effect on the number of firms is ambiguous.*

A transition from monopolistic to perfect competition can be made by setting fixed costs $f=0$ and removing product differentiation setting $\alpha=1$. This increases the tightness ratio and vacancies and decreases unemployment. By implication unemployment is lower under perfect competition than under monopolistic competition.[xvii] In Blanchard and Kiyotaki (1987) this result is achieved by an aggregate demand externality based on real balances in the utility function and endogenous labour supply.

Integrating two identical economies doubles L. This leaves the tightness ratio, the rates of unemployment and vacancies unchanged. The number of firms and varieties is doubled as in the Dixit-Stiglitz model and so are the number of unemployed workers and vacant jobs, uL and vL.

Proposition 7: *Doubling the size of the market doubles the number of firms, the number of unemployed people and the number of vacant jobs, but leaves the rate of unemployment unchanged. Doubling the number of varieties is a gain from trade and integration.*

Matusz (1996 and 1998) finds that the rate of unemployment *does* change through market size effects of international trade: He combines the Dixit-Stiglitz model with an efficiency wage model using a non-shirking constraint to find the result. Matusz (1996) uses the same set up for differentiated intermediates. In his models an increase in market size increases either the productivity of the final output or the variety of consumer goods allowing for a relaxation of the efficiency-wage constraint and a reduction of the rate of unemployment. This result may vanish when there is no non-shirking constraint as in our model. Two other channels - discussed verbally by Matusz (1996) – which might in principal affect the rate of unemployment, are also absent here. First, the price elasticity of demand in his model as in ours is independent of the number of firms. If it were dependent on the number of firms as it is in models with strategic interaction, entry induced by larger market size might affect firms' size and decrease average cost, which might be (similar to) a productivity effect. However, it needs a proof to see the interaction of market size, firms' size, productivity and unemployment. Second, if there were an impact of more variety on the search intensity, the rate of unemployment might be affected. In our set up following Pissarides (1990) this effect is absent because the bargaining is on income and not on utility and the utility function and the Nash-product are specified independently of each other. I agree with Matusz that it is an empirical question whether or not these channels matter quantitatively.

All changes in parameters inducing the above comparative static results have to be interpreted as stemming from perfectly non-anticipated shocks that are expected to be permanent with probability one because the Pissarides part of the model uses

steady-state present values. However, Bean and Pissarides (1993) show that it can be adjusted for use on short-run issues as well.

The economic mechanism (in the sense of finding the solution of the model) of the comparative-static changes as summarised in Figure 1 always goes from the exogenous change to its impact on real wages, hiring costs and the tightness ratio and from there to the rate of unemployment and the number of firms.

This mechanism implies that aggregate output is affected via the number of firms and not via firm size - unless variable cost parameters or the love-of-variety parameter is changed. Changes in the fixed cost parameter have no impact on aggregate output as can be seen from multiplying the solutions for *n* and *x* according to equation (6) and (2') respectively. Aggregate output being mainly affected via the number of firms is in line with the zero-profit condition and the endogenous number of firms and variants.[xviii]

The results of this section are summarized in Tables 2 and 3.

Table 2 ABOUT HERE

The first four columns of table 2 clearly show that Pissarides' results carry over to our model. The fifth column adds results concerning the number of firms added by this model. The sixth column indicates that firm size is independent of labour market parameters in this model.[xix] Finally, the unemployment insurance premium has the same sign as the unemployment rate.

TABLE 3 ABOUT HERE

The columns 5-7 of table 3 are exactly the same as in Dixit-Stiglitz. The first three columns summarize the effects of goods market parameters from the Dixit-Stiglitz model on the labour market variables – these are new results. The column concerning the number of firms has different results from Dixit-Stiglitz concerning marginal costs and marginal revenue because the impact via the unemployment rate changes their results as explained in detail above. The last column adds results from the unemployment insurance premium *t*, which again follows those of the unemployment rate *u*.

5. Summary and conclusion

Linking Pissarides' (1990) search theory of unemployment to the Dixit-Stiglitz (1977) model rather than to the neo-classical production function yields five groups of results. First, the rate of unemployment is linked to the number of firms. Every variable that has an impact on the rate of unemployment also has an impact on the number of firms. Thus, aggregate output is affected via the number firms and not the size of firms unless variable cost and love-of-variety parameters are changed.

Second, the introduction of love-of-variety preferences has an impact as well. A decrease in the love-of-variety parameter, α, decreases the elasticity of substitution and competition, decreases marginal revenue and the tightness ratio, increases unemployment, decreases vacancies and wages and - deviating from the Dixit-Stiglitz model - has an ambiguous effect on the number of firms. It also decreases the size of the firms. By implication employment is higher under perfect than under monopolistic competition.

Third, other comparative static results of Pissarides' model and the model of Dixit and Stiglitz survive. This latter result is mainly due to the fact that hiring costs are added to real wages and therefore the equilibrium quantity is independent of labour market variables. This is similar to the independence of the capital-labour ratio in Pissarides' model from labour market variables. By implication the introduction of hiring costs in both cases decreases wages received by workers.

Four, unlike earlier monopolistic competition models real wages have some flexibility because of the hiring costs depending on the rate of tightness. The model allows for a negative or a positive relation between real wages and the rate of unemployment. The negative relation occurs provided the shocks come from interest and separation rates, the CES parameter or the marginal cost parameter. The positive relation occurs when there is variation in bargaining power and benefits.

Finally, the fixed cost parameter has no impact on the solution for the tightness ratio and other labour market variables and influences only the size and number of firms, but not aggregate output. Doubling market size doubles the number of firms, and the *number* but not the *rate* of unemployed workers and vacant jobs. It increases the gains from trade only via variety.

The Pissarides-Dixit-Stiglitz model of this paper has the desirable properties of both its underlying models, and also some which have been stated as desirable in macroeconomics. The combination of the basic models yields some new results as well. Moreover, it is simple enough to allow for extensions in many directions.

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Table 1 Theories of unemployment and imperfect competition.

Imperfect competition:	*Dixit-Stiglitz*	*others*
Unemployment theory		
Pessimistic expectations	Weitzman 1985 Dutt/Sen 1997	Weitzman 1982
Effort function	Weinrich 1993	
Shapiro-Stiglitz	Matusz 1996, 1998	
Dual labour market	de Groot and v.Schaik (1998, 2000), de Groot (2001)	
Union bargaining		Bräuninger (2000)
Search models	This paper	Aghion/Howitt 1998 Bean/Pissarides 1993

Table 2 Comparative static results of labour market parameters

Effect on → *from increases of ↓*	*tightness θ*	*unempl. u.*	*vac. v*	*wage w*	*firms n*	*size x*	*premium t*
Bargaining power β	-	+	-	+	-	0	+
Hiring costs γ	-	+	-	-	-	0	+
Interest rate r	-	+	-	-	-	0	+
Benefit z	-	+	-	+	-	0	+
Separation rate s	-	+	-	-	-	0	+

Table 3 Comparative static results of goods market parameters

Effect on →tightness θ, unempl. u, vac. v, firms n, wage w, size x, outp. nx, prem t from increases of ↓								
Fixed costs f	0	0	0	-	0	+	0	0
Marginal costs a	-	+	-	-	-	-	-	+
Marginal revenue α	+	-	+	?	+	+	+	-
Country size L	0	0	0	+	0	0	+	0.

List of symbols

α	*marginal revenue, CES parameter*
a	*marginal costs*
β	*bargaining power*
γ	*hiring costs*
f	*fixed costs*
L	*country size*
n	*number of firms*
nx	*aggregate output*
r	*interest rate*
s	*separation rate*
t	*unemployment insurance premium or tax*
θ	*tightness ratio v/u*
u	*unemployment rate.*
v	*vacancy rate*
w	*wage*
x	*firm size*
y	*utility*
z	*unemployment benefit*

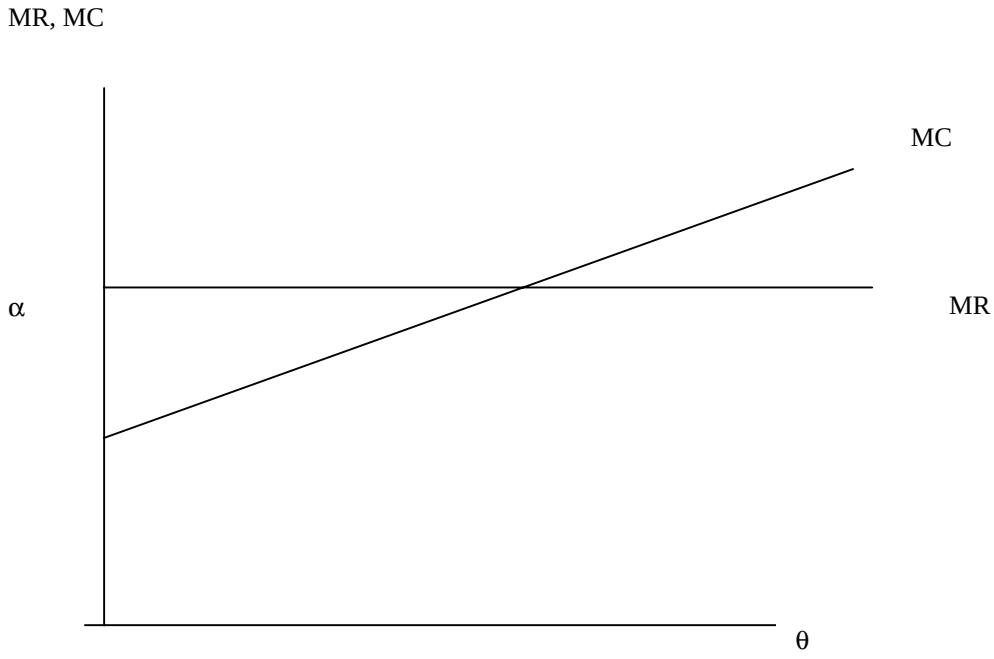

Figure 2. Marginal revenue, MR at value α, is equal to marginal cost, MC. The intersection of both lines determines the tightness ratio θ.

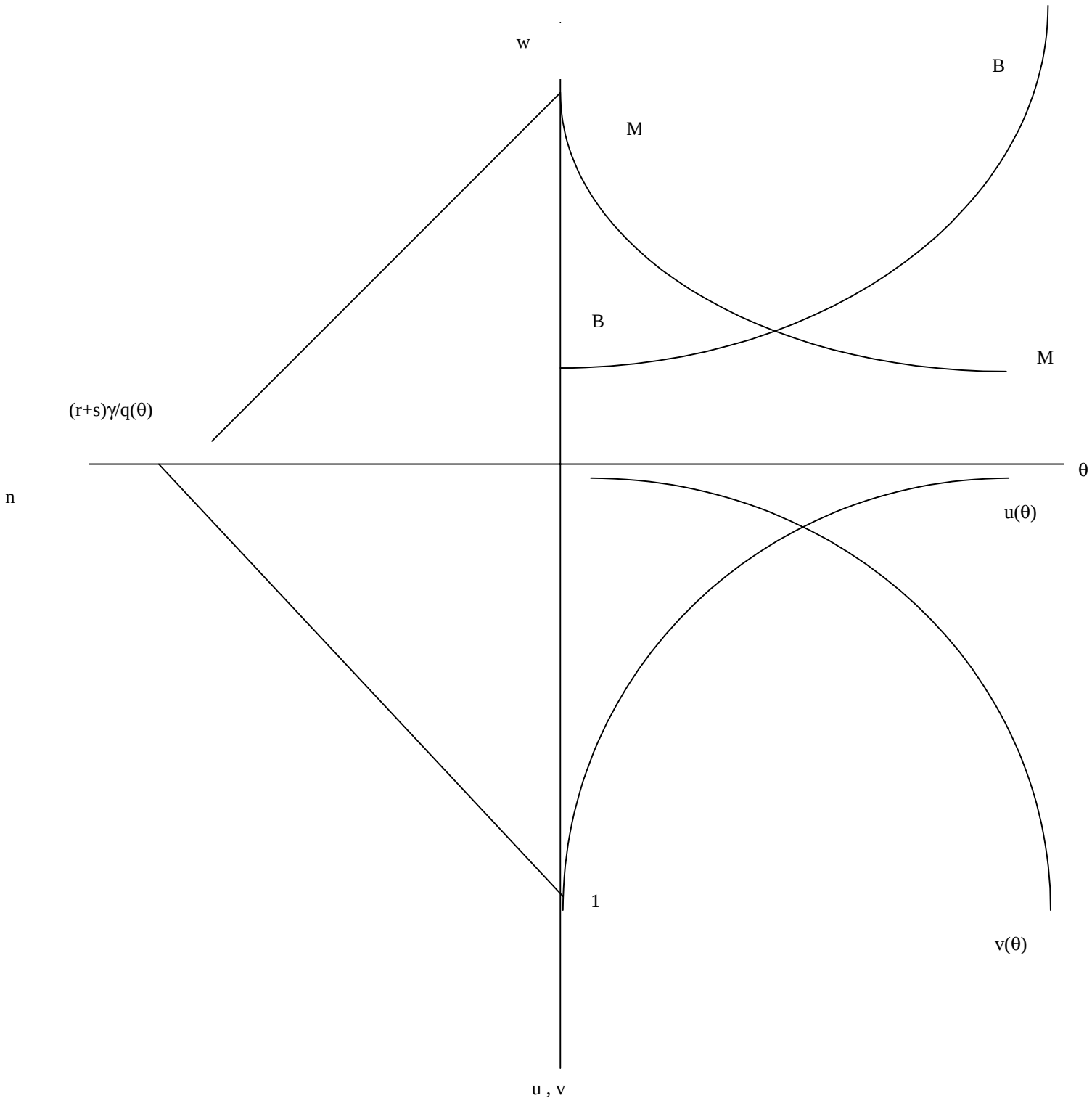

Figure 1. The wage bargaining result, BB, and the profit maximising real wage, MM, determine the real wage and the tightness ratio in the upper right quadrant. This implies a solution for the unemployment rate u and vacancies v in the lower right quadrant. The unemployment rate u determines how many firms can be in the market, in the lower left quadrant. Each result for wages implies a result for hiring costs in the upper left quadrant.

[i] See below
[ii] Subsections are titled as in Pissarides 1990. The search part is explained in greater detail there.
[iii] Pissarides (1998, p.167, footnote 15) refers to estimates of the matching function using a Cob-Douglas functional form, which justifies the assumptions made in the text.
[iv] By implication we only consider the case of a large number of firms in which no strategic behaviour takes place.
[v] In Pissarides (1990) this leads to the zero-profit condition $f(k) - (r+\delta)k - w - (r + s)\gamma/q(\theta) = 0$. Here f(k) is the output per unit of labour and δ is the rate of depreciation.

[vi] Shapiro and Stiglitz (1984, p.435, fn. 5) also follow this procedure.
[vii] As in Pissarides (1990, equation 1.14) this is profits gross of expected hiring costs.
[viii] This result corresponds to equation 1.18 in Pissarides 1990. Note that with $\beta=1$, the negotiation result would require $V=J=\gamma/q=\gamma/(m/v)=0$, which could only hold for $v=0$ without additional assumptions on the matching function. However, with $v=0$ we also have $\theta=0$ and therefore no vacancies and hiring costs. Equation (5) would imply that wages equal revenue per worker.
[ix] This result corresponds to equation 1.19 in Pissarides 1990.
[x] The equation follows from this model but can also be derived from equations 1.20b and c in Pissarides (1990).
[xi] To get a negative second derivative of the MC curve it is sufficient to assume that the matching function is of the Cobb-Douglas type.
[xii] The impact on vacancies remains unclear here. Pissarides' result is achieved if the direct effect of an increase of *s* dominates.
[xiii] It is the formal analogue to the share of capital for a neo-classical production function $f(k)$.
[xiv] Using the properties of the matching function presented on the first page and Euler's theorem we get (with lower indices indicating partial derivatives):

$$-q_\theta \frac{\theta}{q} = \frac{m_1 \frac{1}{\theta}}{m_1 \frac{1}{\theta} + m_2} < 1$$

[xv] Effects of changes in fixed costs may be quite different in endogenous growth models. See de Groot (2000)
[xvi] This result can also be found in the quality ladders literature (see Boone 2000, Aghion and Howitt 1998).
[xvii] The number and size of firms then become indeterminate according to equations (2') and (6), whereas aggregate output becomes nx = (1-u)L/a.
[xviii] As capital is absent in this simple model there is no role for capacity utilisation in the change of aggregate output. In more complex models these relation are likely to be less simplistic.
[xix] Under the assumption of efficiency wages this may be different. It is different in de Groot and van Schaik (1998), but not in Matusz (1996, 1998). This depends on the details of their modeling of efficiency wages.

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