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How to Sow and Reap as You Go: a Simple Model of Cyclical Endogenous Growth

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(MERIT, Maastricht, September 2003)

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Abstract

In this paper, we present a simple endogenous growth model that allows for the occurrence of innovations that can develop into General Purpose Technologies (GPTs), which are the result of basic R&D. The model incorporates the main features of the Romer (1990) model and the Aghion and Howitt (1992) model by using multi-level Ethier functions on the one hand, and Poisson processes to describe the arrival of innovations produced by performing basic R&D and applied R&D. Through basic R&D the core of a potentially new GPT enters the economic system. This core offers the possibility for further expansion of the potential GPT through applied R&D by adding peripherals to this core. The characteristics of the new potential GPT that is represented by the core are randomly distributed. These characteristics include intrinsic profitability, scope for expansion, as well as R&D opportunities and efficiency of the corresponding applied R&D process. By using some illustrative simulations with the model, we show that the arrival of a successful GPT does indeed bring about a reallocation of R&D activities towards applied R&D, thus postponing the moment of arrival of the next GPT. Meanwhile, applied R&D raises the productivity of the GPT as a whole. But the profitability of finding the next/marginal peripheral falls in the process. This fall in marginal profits diminishes the incentives to engage in further applied R&D and increases the incentives to move into basic R&D activities again. Thus, we obtain a cyclical pattern in output growth that is not only partly driven by the arrival of the new potential GPTs but also by the continuing development of existing GPTs in the absence of the arrival of new ones. In periods that do not give rise to the arrival of new successful GPTs we find instances of alternating expansions of existing GPTs that have the character of a GPT-race.

Keywords: General purpose technologies, Endogenous growth, Basic R&D, Applied R&D

JEL classification: O31; O32; O41

1. Introduction

Historical evidence indicates that neither production nor technological progress is a smooth process.[1] This notion of *technological waves* has first been advanced by Schumpeter (1939). He proposed that drastic technological innovation might be the cause of long-run cycles by involving discontinuities in the process of technological innovation.

When we hear about innovations, the common vision, that comes to mind, is that of drastic innovations. *Drastic innovations* are radically new ideas that are reached after deliberate efforts at combining previously unrelated ideas. Therefore, they have no obvious antecedents. This phenomenon has been described as a "new combination" in Schumpeter (1934), "basic innovation" in Mensch (1979), "macro-innovation" in Mokyr (1990), and as "fundamental innovation" in Aghion and Howitt (1998). Drastic innovations are by nature associated with high risks and costs, but usually with the possibility of significant financial returns. The entrepreneur is the main agent in this process and his unique motive is commercial success. In drastic innovations, each new generation good is so much better than its predecessors that the latest innovator is hardly constrained by potential competition from owners of previous patents. This latest innovator is, then, a monopolist and he sets the maximum price that gives the previous innovators non-positive profits and satisfies all the demand at that price, leaving none to the previous innovators. From a technological point of view, however, the most important characteristic of a drastic innovation is that it opens up new fields of technological development opportunities. The concept is, therefore, closely related to what Bresnahan and Trajtenberg (1995) and Helpman (1998) refer to as "General Purpose Technology" (GPT). And the concept is, moreover, closely related to what Aghion and Howitt (1992 and 1998) refer to as quality-improving innovation.

The Aghion and Howitt (1992 and 1998 ch. 2) models start from the assumptions that innovations are drastic and technical change is completely embodied in new intermediate goods that use only the latest technology. Therefore, technical change takes the form of quality-improving innovation (vertical product differentiation), and only one intermediate good is used at a certain point in time. Succeeding intermediate goods embody quality improvement, which render their predecessors obsolete. This is the "creative destruction" effect pointed out by Schumpeter (1934). One of the most interesting features of the Aghion

[1] See on this subject, for instance, Olsson (2001) and Gordon (2000).

and Howitt model is that the uncertainty of the research process implies the possibility of cyclical growth patterns.

We usually contrast drastic innovation with incremental innovations. *Incremental innovations* are small, non-revolutionary changes in technology that are carried out by profit-oriented entrepreneurs. These incremental innovations refine existing knowledge in a predictable fashion and are generated when entrepreneurs combine older insights that are closely related from a technological point of view. This phenomenon has, also, been described as “micro-innovations” in Mokyr (1990), “refinements” in Jovanovic and Rob (1990), and “secondary innovations” in Aghion and Howitt (1998, ch. 6). The costs and risks associated with these incremental innovations activity are relatively low. And the boundaries to incremental innovation are set by the prevailing technological family, which defines the opportunities of technological research at some point in time. As Dosi (1988), we can say that incremental innovations are highly path dependent, following specific technological trajectories. The most important characteristic of an incremental innovation is, therefore, that it belongs to a technology family. The concept is closely related to Romer (1990), because this model uses the concept of intermediate goods that are built in accordance with specific blueprints. These intermediate goods could be seen as incremental innovations, because the Romer model assumes that technical change is embodied in the ensemble of intermediate goods rather than in only one new intermediate good as in Aghion and Howitt model. This technical change takes the form of variety-improving innovation (horizontal product differentiation), and the productivity of all intermediate goods is the same because Romer has assumed that intermediate goods are completely symmetric with respect to their contribution to output.[2] Because of this symmetry between intermediates, technical change only increases the number of all intermediate goods used in producing output. But by doing so, technical change does generate productivity growth through improved division of production tasks between intermediates. The main idea of the Romer model is that an increase in the number of intermediates facilitates the division of production activities in a way that is comparable to the notion of Smithsonian labour division, because the technical change is of an organizational nature.

In order to introduce vertical product differentiation to be able to cover the arrival of drastic innovations and their subsequent expansion into GPTs, we therefore have to include

[2] On this subject, chapter 6 of Barro and Sala-I-Martin (1995) considers models in which technological progress shows up as an expansion of the number of varieties of products.

asymmetries in the contribution of intermediates to final output. We do this by respecifying the Ethier production structure into a multi-level one, where the upper-level combines GPTs to generate output, and where the lower levels describe how the components of a GPT define and drive the further development of a GPT.

This asymmetry between intermediates introduces qualities differences between intermediates as in van Zon and Yetkiner (2003). The conceptual organization of the production structure is therefore as follows: GPTs lead to growth due to horizontal (and symmetric) GPT differentiation on the one hand, while further expansion of a GPT by adding more components raises the productivity of the corresponding GPT again, thus effectively breaking the symmetry between GPTs again. Neither innovation is so drastic that it does indeed completely drive out other ones, however they can be asymptotically drastic.[3] We are calling the drastic innovation born from basic R&D "the technology core" that gives birth to a new technology and enables the subsequent creation of a whole new GPT or even technological paradigm. The evolution of a new technology occurs as the result of incremental innovations that build on the core and that are horizontally differentiated. These incremental innovations belong to the same "technology family" as the core.[4] We are calling these incremental innovations that are born from applied R&D: "the technology peripherals". By assumption, the contribution of each peripheral to the overall technology decreases with the time of arrival of a peripheral, i.e. the most important peripherals are invented first.

In the rest of this paper, we will be using the terms "core" and "peripherals" of a technology, instead of "drastic innovation" and "incremental innovation". This terminology allows us to think of every technology as being composed of first a core that could be seen as a potential GPT, and secondly peripherals that resemble intermediate goods as in Romer (1990) and van Zon and Yetkiner (2003).

Our model, then, takes inspiration from the Aghion and Howitt (1992 and 1998 ch. 2) model and the Romer (1990) model. As in Romer (1990) we have many technologies being used at the same time, while as in van Zon and Yetkiner (2003), we allow for qualitative differences between technologies and between technology peripherals inside a technology. In

[3] A GPT is therefore not drastic in the usual sense, and we would have preferred to use the term "technology family" for this situation, but for the name inflation this entails. The reason for this is that the term "family" refers to a technology group, and within a family we could have different generations of this technology.
[4] Just as Dosi (1988), when the new paradigm is generally accepted, a period of normal incremental innovations resumes along the trajectories defined by the new paradigm.

our model, therefore, productivity growth at the aggregate level is the result of both love of varieties and 'inverted quality ladders'.

Further details of our model are as follows: As in the Romer model, a representative firm in the final goods sector produces output by using labour and a set of production technologies combined in an Ethier function (Ethier (1982)).[5] Each technology acts as a composite input made of different components belonging to that technology.

In addition, the model consists of one basic R&D sector that generates each technology, and as many applied R&D sectors, as there are technologies, that develop these technologies by generating peripherals for each technology. The production function used to define the technology is a CES function of components, which are the core and peripherals. Contrary to Romer (1990), the peripheral contribution is no longer symmetric in our model. Instead, a technology is assumed to consist of a core and of peripherals that contribute less and less to the productivity of the technology as a whole. But as in Romer (1990), the monopoly profits of the intermediate sectors are transferred to R&D workers in the form of wages (and profits unlike Romer (1990)), and researchers will continue to exploit positive profit opportunities of a current technology by making new peripherals. When peripheral profit opportunities become exhausted, it becomes more yielding to invest in a new technology. Researchers then switch to work on the next technology core. However, this switch is not complete, i.e. there is no bang-bang behaviour regarding the distribution of R&D workers over applied and basic R&D. The quest for technology peripherals is resumed within the new technology and continues until profit opportunities fall below that of other research alternatives, among which basic R&D. Thus, technological change and economic development follows a cyclical pattern.

The model contributes to the growth literature in several ways: First, it introduces asymmetries in the intermediate goods market, which is rarely done in the literature.[6] This paper shows that asymmetric profit opportunities in intermediate goods sectors is more than a mere detail. Indeed, as Yetkiner (2003) points out, falling profits provide the real incentives to R&D to find the next completely new technology. Secondly, given the significant effects of technological change on economic growth, a better understanding of the reasons behind the cyclical evolution of output and technology is important from a policy perspective. In

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[5] Ethier (1982) defined an extension of the Cobb-Douglas production function. This Ethier's function results of two functions combination a Cobb Douglas and a CES. In the context of technological change and economic growth, Romer (1990) used Ethier's production function with a variety of productive inputs.

[6] To our knowledge, van Zon and Yetkiner (2003) is the only work studying asymmetric intermediate sectors in an endogenous technological framework.

particular, smoothing the cyclical evolution may improve the long run performance of an economy (Yetkiner (2003)). Third, our model elaborates on the role of basic and applied R&D mechanisms in the growth process. It shows that the influence of these two R&D types on the long-run growth process is significantly different.

The paper is organized as follows: In section 2, we provide a brief review of some alternative approaches to modelling technological waves. In section 3, we explain the main features of our model. Section 4 contains the results. Finally, section 5 provides some concluding remarks.

2. Alternative Approaches regarding Technological Waves

The literature on endogenous growth with technological innovations describes two kinds of models. The first kind represents horizontal product differentiation, i.e. a variety-expanding innovations. These models have for instance been developed by Romer (1990), Grossman and Helpman (1991, ch. 3), Barro and Sala-I-Martin (1995, ch. 6). Secondly, Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991, ch. 4), Aghion and Howitt (1992), Barro and Sala-i-Martin (1995, ch. 7), and Aghion and Howitt (1998, ch. 3) attribute economic growth to vertical product differentiation, i.e. to quality-improving innovations. Technological progress in both kinds of models is based on R&D, and the role of imperfect competition is to provide the incentives to engage in R&D. In R&D-based endogenous growth models, the pace of long-run growth is solely determined by the number of researchers, respectively by the level of research expenditure. Consequently, according to these models, subsidization of research leads unambiguously to a higher long-run growth rate.[7]

Surprisingly, drastic innovations have not received much attention in the growth literature. Precursory contributions include Jovanovic and Rob (1990) and Cheng and Dinopoulos (1992) who try to generate Schumpeterian waves based on the dichotomy between fundamental and secondary innovations, with each fundamental innovation being followed by a sequence of more and more incremental innovations. Of particular interest as a macroeconomic model is Cheng and Dinopoulos (1992), in which Schumpeterian waves are obtained as a unique, non steady-state, equilibrium solution, in which the current flow of monopoly profits follows a cyclical evolution: “Because the economy’s wealth is equal to the

[7] But not necessarily to higher welfare, because a reallocation of resources toward R&D goes at the expense of current production, hence current consumption and investment opportunities.

discounted present value of aggregate monopoly profits, fluctuations in profits generate procyclical fluctuations in wealth, the interest factor, consumption […] and aggregate R&D investment." Solow (1997), also, follows an out of equilibrium approach by introducing in the Arrow (1962) learning by doing model the notion of discrete innovation. The learning by doing cannot sustain the growth rate indefinitely, however. That is why the introduction of discrete innovations as new objects of learning is necessary. And so a non-linearity appears in the learning function. The transition between two innovations is due to the learning process and does not lead to a slow down of the growth rate. The dichotomy between fundamental and secondary innovations goes further with the Aghion and Howitt (1998a) approach. In their chapter 6 called "Learning by doing and secondary innovations", Aghion and Howitt consider heterogeneity in the innovation process: the fundamental innovation comes from the research process and the secondary innovation from the production process.[8] They introduce a research capital concept, which is the mass of researchers, or workers who have chosen to do the research. They explain a growth rate slowdown by the fact that the work force switches from the production sector to the research sector.

Another approach is the "general purpose technologies" (GPTs) approach initiated by David (1990) and especially Bresnahan and Trajtenberg (1995).[9] This approach focuses on the temporary cyclical effects that may be created by new technological paradigms at the beginning of their introduction to the economy. Each GPT raises output and productivity in the long run, it can also cause cyclical fluctuations while the economy adjusts to it. As David (1990) and Lipsey and Bekar (1995) have argued, such GPTs require costly restructuring and adjustment to take place, and there is no reason to expect this process to proceed smoothly over time. These cyclical downturns may be the price that society needs to pay in order to implement the GPTs that deliver the long-run growth.[10] These technological wave approaches can be summarizing by the title of Helpman and Trajtenberg (1994): "A Time to Sow and a Time to Reap". They develop a simple theoretical model in order to study the macroeconomic consequences of GPTs. In this model, monopolistic firms develop intermediate inputs and sell them to manufacturers of a final consumer good. Technological change is exogenous and appears at fixed time intervals. They find that each time an innovation occurs it creates a

[8] Aghion and Howitt (1998a) pages 173 to 195.
[9] Examples of GPTs that have affected the entire economic system include the steam engine, the electric dynamo, the laser, and the computer. The future GPT could be nanotechnology.
[10] Thus, contrary to the predictions of real business cycle theory, the initial effect of a "positive technology shock" may not be to raise output, productivity and employment but to reduce them.

cycle: The first phase is the “time to sow”, when the resources are used to the development of complementary factors, which are profitable to the new innovation. In this phase, the growth rate slows down. The second phase is the “time to reap”, when enough complementary factors have been creating, it is useful to change the production process and to actually use the higher production potential of the new GPT. In this phase, the growth rate increases.

Aghion and Howitt follow the Schumpeterian idea of technological fluctuations and the Kondratieff idea of long cycles to develop a number of models that are based on this notion. In Aghion and Howitt (1998b), the authors retain Helpman and Trajtenberg’s framework and introduce externalities based on collective learning. This collective learning process benefits from and is necessary for the adoption of a new technology, while in turn it facilitates the adoption of still newer technologies. We could say that it is part of the diffusion process of a new technology. The macroeconomic effect is a slowdown after the arrival of the new technology.

The main aim of the literature outlined above is to emphasize the difference between drastic technologies and incremental technological changes in terms of their growth implications. Currently, the focus seems to be on whether an economy experiences a slowdown at the beginning of a new technological change due to reallocation of resources from the old to the new sectors or not.[11]

3. The Model

3.1 Introduction and overview

GPTs are ex post mental constructs. During the innovation process, the actual pervasiveness of an innovation when and if it arrives can at best only be guessed at. This pervasiveness depends on the way in which an innovation can replace some productive function that was previously executed by other means, or on the way in which an innovation gives rise to the execution of functions that did not previously exist but have a high value of their own. Hence, if one wants to model the arrival of GPTs and their further development over time, one is logically forced to express the GPTs potential for productivity growth and expansion in terms of broad intrinsic characteristics, rather than stating up front that a specific

[11] See several chapters in Helpman (1998).

innovation REPRESENTS a GPT by assumption.[12] In that sense all results of the basic innovation process are potential GPTs: it's only the extent of their use and their scope for further extension that change a potential GPT into a real one.

The model we are about to present underlines these principles: GPTs all start off on an equal footing, i.e. they consist of a core invention that has a productive use that defines the nature of the GPT and its field of application. "Peripherals" can then be added to this core that increase the overall productivity of the GPT consisting of the ensemble of a core and its corresponding peripherals. The set of the core and its peripherals will further be called the components of a GPT. In such a setting, it seems logical to us that, as a rule, the peripherals that would contribute the most to the productivity of a GPT as a whole are added first, and more marginal peripherals are added later. This reflects our intuition that the scope for further expansion of the overall productivity of a GPT declines with the addition of still further components. Then again, love of variety, i.e. increased within GPT specialisation due to the increase in the number of components of a GPT, may counter this negative effect of the expansion of a GPT on its overall productivity. We conclude from this observation, that in order to properly describe these characteristics, we should use a specification of a GPT that allows for both love of variety (i.e. increasing returns due to increasing varieties) and decreasing returns to increasing varieties. We will implement this by using nested Ethier aggregator functions.[13]

In addition to this, we account for the probabilistic nature of the innovation process, including the question whether some innovation has the potential to become a GPT, by using Poisson processes to describe the arrival of innovations, à la Aghion and Howitt (1992,1998). Furthermore, we randomly draw the intrinsic characteristics of the core of a potential GPT that matter for its future prospects for turning into a real GPT. So we do not know on beforehand whether a basic innovation will turn out to be the core of a real GPT with a large extension or of a GPT that does not have any peripherals at all (and that is therefore not a real GPT ex post, but a "failed GPT" instead).

We distinguish between two R&D processes with different functions. The growth of a potential GPT by adding peripherals is done through applied R&D, whereas the core of a potential GPT comes from basic R&D. Both types of R&D are explicitly accounted for in the

[12] This has been the usual modelling approach up to now; again see Helpman (1998) and Helpman and Trajtenberg (1994), Bresnahan and Trajtenberg (1995).

[13] Ethier (1982) defined an extension of the Cobb-Douglas production function. In the context of technological change and economic growth, Romer (1990) used Ethier's production function with a variety of productive inputs.

model, as well as their relative size that is driven by profit incentives, very much as in Romer (1990) and in Aghion and Howitt (1992). We will show that even with the same specification of the R&D production processes, cyclical growth patterns will occur, where periods of relatively fast growth are followed by periods of slow growth, but not negative growth.

In addition to this, we slightly depart from the usual specification of the R&D production function, where R&D output is directly proportional to current R&D input. We introduce decreasing returns to current R&D input, as in van Zon (2001), in order to avoid the bang-bang R&D labour reallocations that are implied by the use of such proportional production functions in a perfect competition context (see, for instance, Yetkiner (2003)). Instead, what we will show is that basic R&D and applied R&D are processes that do indeed depend on each other, but that do not completely cannibalise each other although some nibbling or even a bite or two can occur.[14]

In the GPT literature, the slump in the growth of output generated by the arrival of a GPT is due to the fact that labour resources are drawn away from directly productive uses and are used instead for the further development of a GPT.[15] In reality the labour market is more segmented, and one would expect little or no production labour to flow into R&D sectors, certainly not of this production labour supplies what Romer (1990) refers to as "eye hand coordination". Research labour flowing into the final output sector is another matter. Presumably researchers are also capable of a satisfactory degree of "eye hand coordination", or white-collar activities. Nonetheless, we will take the position that there are two separate labour markets: one for production labour, and one for research labour, just to make the point that the cyclical development of output is not necessarily driven by people changing their overalls and putting on their laboratory kit, and the other way around.

The qualitative results we expect from the modelling set-up outlined here and explained in more detail below, is that for given available labour resources, labour productivity can grow through the arrival of basic innovations that function as the core of a potential GPT that can turn into a real GPT if only the incentives to add more and more peripherals to the core are right. These incentives are driven by the way in which the GPT is used in final output production, but it is also driven by the scope for and costs of further expansion of the GPT through applied R&D. In general however, we would expect that the core arrival injects new profit opportunities into the economy that can be realised by inventing peripherals to the new core. In the end the incentives become less, because of the decrease in the marginal

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[14] Some nibbling will always take place because of the Inada conditions.
[15] See Helpman (1998).

contribution of the latest peripheral to the productivity of the GPT as a whole, and the relative incentive to engage in basic R&D again becomes stronger. We would expect therefore an arrival of a core to be followed by a spur of applied R&D activity, thus lowering the arrival rate of new GPTs. Depending on the rate of decrease of the marginal productivity of adding a peripheral versus the rate of increase of the overall productivity through love of variety, the growth rate may increase for a while before levelling off and falling again, or it may instantaneously increase and fall immediately but gently or even abruptly. Such developments will be sketched and explained in more detail in section 4, where we provide some illustrative model simulations. The remainder of this section is devoted to a detailed description of the different parts of the model.

3.2 Final output production

Final output is produced using a Cobb-Douglas production function that uses raw labour as an input, next to a capital aggregate that is assumed to have a GPT inner-structure given by a symmetric CES function. This combination effectively results in an Ethier function for final output, very much as in Romer (1990), but without the human capital present there:

$$Y = L_y^{1-\alpha} \cdot K_e^{\alpha} \qquad 0 < \alpha < 1 \tag{1}$$

where Y represents final output, Ly production labour, and Ke is the effective capital aggregate, further given by:

$$K_e = \left(\sum_{j=1}^{A} z_j^{\alpha} \right)^{1/\alpha} \tag{2}$$

where A is the number of GPTs currently active, and j indexes those active GPTs. z_j represents the "volume" of GPT j. Obviously, equation (2) is a CES function with a symmetric contribution of all factors z_j to the level of effective capital K_e. The final good sector is perfectly competitive, and because the production function is homogeneous of degree one, final good production can be described as the production by a representative firm.

An important aspect of the model is that inputs are not perfect substitutes: GPTs are better substitutes for each other than labour and the effective capital aggregate are. Equations (1) and (2) provide a very simple structure, that can easily be generalised.[16] Indeed, three obvious generalisations of (2) spring to mind:

- a Mukerji (1963) function instead of a CES, so that some GPTs may be more substitutable for other GPTs than others;
- a CES but with different contributions of each GPT to final output;
- a CES with a within capital complex Ke elasticity of substitution that is not necessarily equal to $1/(1-\alpha)$.

However, our immediate aim is to show what difference the multi-level organisation of the production process and its interpretation in terms of GPT and their composing components makes for the way in which growth may take place. That is why we use “standard” growth model components from the toolbox provided by the models of Romer (1990) and Aghion and Howitt (1992). It is only the “new combination” that we make of these growth-tools that we want to investigate in terms of its impact on growth patterns.

3.3 The GPT inner-structure

A GPT is assumed to consist of a core and of peripherals that contribute less and less to the productivity of the GPT as a whole. Because we want to stick as closely as possible to the original Romer (1990) model, because of its love of variety features, an obvious candidate to describe the inner-structure of z_j is again a CES function:

$$z_j = \left(\sum_{i=0}^{A_j} c_{i,j} \cdot x_{i,j}^{\beta_j} \right)^{1/\beta_j} \tag{3}$$

where $x_{i,j}$ for i>0 represents the i-th peripheral of GPT j and $x_{0,j}$ represents the size of the core of GPT j.[17] $c_{i,\ j}$ are the standard distribution parameters one normally uses with a CES function, while $1/(1-\beta_j)$ is the elasticity of substitution between all the components of GPT j. A_j is the number of peripherals belonging to GPT j. In the very early stages of a GPT, A_j can

[16] We will not do this here, because the behaviour we have obtained with the simple structure described here is already interesting enough by itself, as we will show in section 4 later on.

[17] Peripherals can be vaguely associated with “innovational complementarity” character of GPTs as advanced by Bresnahan and Trajtenberg (1995), that is “the productivity of R&D in a downstream sector increases as a consequence of innovation in the GPT technology”. In fact the notion of complementarity hides the interpretation of productivity increases due to improved division of production tasks between intermediates, whereas formally individual intermediates are direct substitute for each other.

actually be equal to zero, and in fact, as we will illustrate in the next section, even in the medium and long run, *Aj* can remain equal to zero, thus underlining the ex post character of what we consider to be real GPTs i.e. technologies with a large number of peripherals. For GPTs with a small number of peripherals *Aj,* we will use the term “failed GPTs” from now on.

In order to simplify matters as much as possible, we will make the following assumptions:

$$\beta_j = \alpha \ \forall \ j \tag{4.A}$$

$$c_{i,j} = c_{0,j} \cdot \left(\varsigma_j\right)^i \ , \ \ 0 \ \pi \ \varsigma_j \ \pi \ 1 \tag{4.B}$$

Assumption (4.A) reduces the three-level organisation of the production process effectively to a two-level production function with asymmetric contributions of all components of all GPTs to final output, while (4.B) puts a technically useful structure on the distribution coefficients of the implied aggregate production function. This structure allows us to write the production function in terms of an aggregate of mathematical transformations of the cores of the various GPTs only, so from a practical point of view we can essentially forget about individual peripherals. This is a big technical bonus since we have to deal with a number of “real GPTs” simultaneously. (4.B) states that the CES distribution coefficients are geometrically declining with the peripheral index.

3.4 The demand for GPT components

The demand for GPTs is derived very much as in Romer (1990). Assumption (4.A) and equations (3) and (2) when substituted into (1), give rise to the following inverse demand equations for each individual component, under the assumption of perfect competition in the final output market:

$$x_{i,j} = L_y \cdot \left(\frac{p_{i,j}}{c_{i,j} \cdot \alpha} \right)^{-1/(1-\alpha)} \tag{5}$$

Assuming as in Romer (1990) that the components can be produced using raw capital only, where each component *i* of GPT *j* takes η_j units of raw capital $k_{i,j}$ to create one unit of the component $x_{i,j}$, the marginal production costs are equal to $\eta_j \cdot r$, where r is the interest rate and where we have ignored the depreciation of capital. Because each component has its own market niche (as described by (5)), the profit maximising rental price of each component is easily obtained as:

$$p_{i,j} = r \cdot \eta_j / \alpha \tag{6}$$

which is the familiar Amoroso Robinson condition for profit maximisation under imperfect competition.

Using (6) to obtain total profits per component, we find:

$$\pi_{i,j} = L_y \cdot c_{0,j}^{\;1/(1-\alpha)} \cdot \varsigma_j^{\;i/(1-\alpha)} \cdot (1-\alpha) \cdot \alpha^{(1+\alpha)/(1-\alpha)} \cdot (r \cdot \eta_j)^{-\alpha/(1-\alpha)} \tag{7}$$

where $\pi_{i,j}$ are the profits associated with the *i*-th component of GPT j. Equation (7) has some interesting features. First, profits of a component rise with the overall level of final output production as proxied by L_y. Secondly, they fall with a rise in the production cost of a component (i.e. $r \cdot \eta_j$), while third they fall with the peripheral index i (since $0 \pi \varsigma_j \pi 1$). The latter is one of the most important drivers of the overall behaviour of the model. Note too that for constant values of L_y and r, ex post profit flows are constant too. Under these assumptions, the present value of the profit stream associated with using a peripheral would be given by:

$$PV\pi_{i,j} = \pi_{i,j} / r \tag{8}$$

where $PV\pi_{i,j}$ represents the present value of the profit flows obtained from renting component i of GPT j. As in Romer (1990) and Aghion and Howitt (1992), we assume that these flows are captured by the respective R&D sectors that created the designs for these components.

3.5 R&D activity

The blueprints that are needed to be able to produce GPTs are obtained by the intermediate goods sectors from the R&D sector. We assume that each innovation, whether basic or applied, is the result of innovative activities from labour in that sector. This labour is endowed with the frontier knowledge that is required to do research and can be engaged in basic or applied R&D. The determination of the activity, which the research labour engages in, depends on the relative profitability of both types of R&D. The relative profitability of both types of R&D depends on the profitability of adding peripherals to an already existing technology (for which the core already exists) versus creating a completely new technology (for which a new core is required). As we will show, the profitability of peripheral falls the later it is introduced, so that there is a certain point at which pursuing basic R&D becomes increasingly more profitable than doing applied R&D, and research labour shifts from doing applied research to doing basic research. However, the shift is not complete for reasons made clear further below. Whatever the specific engagement of R&D labour, the output of research labour is always an innovation, which we assume is patented and which serves as an input to the production of composite goods. The costs of producing components of a GPT, therefore, include the costs of getting hold of the patent.

Our motivation behind considering different R&D processes for the case of a peripheral on the one hand and that of a core on the other hand is our perception that the invention of a technology core requires "something more fundamental" than the further development of a technology by adding peripherals. We capture this difference by differentiating their contribution to total production. However, we also feel that finding a (core of a) potential GPT is subject to more uncertainty than finding a peripheral once a new technological "proto-paradigm" has arrived in the form of a potential GPT. We model this by assuming that the R&D process itself is uncertain first because it is not able to predict the arrival of a GPT, and secondly because it is not able to predict the actual characteristics of a potential GPT, that we assume to be known only after the arrival of its core. The expectations one has had about the inherent productivity of the next GPT (i.e. $c_{0,j}$), about the production costs of the peripherals (i.e. $r.\eta_j$), the scope for extension (i.e. ς_j), associated applied research productivity (i.e. δ_j) and research opportunities (i.e. μ_j) will prove to have been too optimistic or too pessimistic if and when the new core arrives.

We distinguish between basic and applied R&D but model both types along the same lines. R&D gives rise to innovations that arrive according to a Poisson probability distribution with arrival parameter λ. According to this distribution, the probability that an event (arrival of an innovation) occurs within T time units from now, is equal to F(T)=1-$e^{-\lambda \cdot T}$.

As in Aghion and Howitt, we assume that the level of R&D activities directly and positively influence the arrival rate of innovations. Unlike Aghion and Howitt (1992), however, we assume that there are decreasing marginal returns to current R&D, giving rise to an effective arrival rate of innovations given by:

$$\lambda_j = \mu_j \cdot R_j^e \tag{9.A}$$

$$R_j^e = \delta_j \cdot R_j^{\ \beta} \tag{9.B}$$

where λ_j is the arrival rate of innovations associated with R&D process j (process 0 is associated with the basic R&D necessary to find the core of the next GPT, whereas j>0 represent the processes necessary to find the next peripheral of GPT j). μ_j is the arrival rate at a unit volume of effective R&D, R^e_j. Equation (9.B) shows how effective R&D uses R&D labour Rj with an average "productivity" equal to δ_j at unit level, and where $0 \pi \beta \pi 1$ ensures that the marginal product of R&D labour falls with the level of R&D input. The latter is another main feature of the model, since it ensures that in combination with (8), the marginal benefits of doing applied R&D and basic R&D are asymptotically falling to zero for increasing levels of R&D inputs. But more importantly, they rise to infinity for levels of R&D inputs that fall asymptotically to zero. The latter ensures, that it will always be profitable to employ a non-zero volume of R&D workers on any project, however bleak the prospects for success may be.

The marginal benefits from doing R&D on the *i*-th peripheral of GPT *j* are now given by:

$$MB_{i,j} = \frac{\partial(PV\pi_{i,j} \cdot \mu_j \cdot \delta_j \cdot R_j^{\ \beta})}{\partial R_j} \tag{10}$$

Due to the free mobility of R&D labour between its various uses, the marginal benefits for different R&D activities should be the same, giving rise to the following optimum ratio of applied R&D versus basic R&D workers:

$$R_j / R_0 = \left(\frac{PV\pi_{Aj_i+1,j} \cdot \mu_j \cdot \delta_j}{PV\pi_{0,A+1} \cdot \mu_0 \cdot \delta_0} \right)^{1/(1-\beta)} \equiv \varphi_j \tag{11}$$

where A_i is the number of peripherals of GPT *j*, and *A* is the total number of active GPTs. Equation (11) shows that higher (expected) profit flows on a peripheral in some GPT will divert R&D resources into further expanding that GPT. Such expansion is promoted by an R&D process that is relatively efficient (high δ_j), or where innovations are relatively easy because of ample "fishing" opportunities (high value of the arrival parameter μ_j). Because total R&D uses exhaust the available supply of R&D workers R, we readily find:

$$R_0 = R/(1+\sum_{j=1}^{A}\varphi_j) \tag{12.A}$$

$$R_j = \varphi_j \cdot R_0 \tag{12.B}$$

A rise in the present value of some expected profit flow will therefore raise the corresponding φ_j and for given *R*, the corresponding value of R_j will unambiguously rise at the expense of other R&D activities, thus leading to a direct trade-off between applied and basic R&D activities, i.e. expansion of existing GPTs versus the creation of new ones.

3.6 Interest rate determination

As in Romer (1990), capital in this model is accumulated consumption foregone. Furthermore, capital is completely putty. The amount of investment is given by a proportional saving function, as for instance in the neo-classical growth model. The interest rate is assumed to adjust instantaneously and continuously until the total demand for physical capital exactly matches the supply of capital in the form of accumulated consumption foregone.

Interestingly enough, this set-up does provide a link, at least in principle, between changes in the saving rate and the rate of technical change, in as far as these changes succeed in changing both available investment-capital and the interest rate. The reason is that a lower supply of financial capital (due to lower savings for instance), raises the interest rate, which in turn depresses the incentives to engage in R&D. In our model, however, relative R&D activities do not depend on the interest rate, whereas total R&D labour is given by assumption. Hence, a decrease in savings would lower the wage rate of R&D workers directly (less demand for R&D labour for the same supply), while the marginal product of labour in the final output sector would also be negatively affected, but less so than the marginal benefits of R&D labour in the R&D sectors.[18] Consequently, a rise in the rate of interest will lower real wages everywhere but more so in R&D activities.

3.7 Model closure

The model is closed by assuming a proportional saving function, giving rise to:

$$\Delta K = s \cdot Y \tag{13}$$

where Δ is the first difference operator, and s is the constant saving rate, and where we have disregarded the depreciation of capital.

3.8 Concluding remarks: Love of variety issues

Equation (13) describes the accumulation of physical capital. However it is the organisation of this capital in the form of different GPTs with differential impacts on effective capital that ultimately results in the growth of capital productivity, hence output itself, for a given amount of labour and in the absence of labour augmenting technical change. This is easy to see, since after some manipulation of the relation between effective capital and the respective sizes of the core, as well as the capital costs of building the core (and the corresponding peripherals), we get:

$$K_e{}^j = K_j \cdot c_{0,j}{}^{1/\alpha} \cdot \{(1-\varsigma_j^{(1+A_j)/(1-\alpha)})/(1-\varsigma_j^{1/(1-\alpha)})\}^{1/\alpha-1} / \eta_j \tag{14}$$

The "effective capital" productivity of "raw" capital depends positively on the size of the contribution of the core (i.e. $c_{0,j}$), positively on the scope for extension of the GPT (i.e. ς_j), negatively on the raw capital cost per unit of effective capital (η_j), but most importantly, positively on the number of peripherals of the GPT, (i.e. A_j). The latter is the love of variety effect implied by the concavity of the effective capital aggregate in its individual components

[18] The marginal product of labour is proportional to output and the partial output elasticity of labour, and inversely proportional to the given number of final output workers. So, as long as the number of workers exceeds the value of the partial output elasticity of labour, the change in the final output wage rate due to a one-unit change in accumulated savings is smaller than the marginal product of capital.

(cf. equation (3)). Love of variety works at two different levels in this model, however, as opposed to Romer (1990). It works at the component level within each individual GPT, but also at the level of the GPTs that are used to produce final output.

4. Results and Policy extensions

4.1 Introduction

The model we have outlined above, will be used for some illustrative simulations. In order to do that we have implemented it in the following way. First we have assumed that basic R&D is intrinsically more uncertain than applied R&D. This results in an allocation of R&D resources between basic and applied R&D activities, where the marginal benefits from doing basic R&D are uncertain, whereas those from applied R&D on active GPT's are not. We model this by assuming that expected profits are based on the expected values of the driving parameters (arrival parameters, R&D productivity parameters, and so on). For the profit flows coming from the invention and subsequent use of new peripherals, we assume that all the defining characteristics are know given the characteristics of the core.

The flow of logic of the model is now as follows. Profitability expectations drive the allocation of R&D workers over their various uses. Draws from the various Poisson distributions for basic R&D but also for applied R&D of the active GPT's lead to a success (a new peripheral for applied R&D, or a new potential GPT for basic R&D) or not. Whatever the exact time of the success, R&D resources are assumed to be allocated for an entire tick of the clock and will not be used for other purposes if and when the innovation happens within a tick of the clock.

The subsequent arrival of a GPT or (set of) peripherals changes the relative profitability of a GPT (hence of expanding all active GPT's) and that of the peripherals making up a GPT. This normally leads to a reallocation of R&D resources in the direction of applied R&D after the moment of invention of the core of a new GPT, unless the further extension of an old and active GPT proves to be more profitable. This could be the case if the scope for extension of the new GPT is very high, or the effective applied R&D productivity is very low. We will show such an instance further below.

If by pure chance a GPT does not arrive for any length of time, applied R&D while expanding exisiting GPT's runs slowly out of gas (i.e. profit opportunities), thus making basic

R&D relatively more attractive, and ensuring (in a probabilistic way) that new expansion possibilities will arise in the future with the timely arrival of a new GPT.

A GPT, if and when it arrives, needs capital to be implemented next to other existing GPT's. If the contribution of the new GPT to final output is non marginal, the interest rate will be driven up, thus crowding out the other GPT's in the process, as well as their expansion possibilities through applied R&D. This switching between different types of R&D smoothes the growth spikes that are inherently tied to the random arrival of new GPT's and new peripherals. Indeed the fact that there are multiple GPT's active at the same time, further smooths the growth process, in that the joint activity on expanding existing GPT's lowers the probability that no peripheral or GPT core will arrive during a tick of the clock. This is by no means just a diversification issue. Due to the assumed decrease in the marginal productivity of R&D labour, having available more GPT expansion possibilities absolutely raises the productivity of the group of R&D workers as a whole, due to the assumed concavity of the R&D production functions.

After the arrival of an innovation, it is active from the next period. If the innovation pertains to the core of a new GPT, new peripherals can be looked for from the next period, and they can be added to the core again after the period of their invention.

4.2 Parameter sets

In order to perform some model simulations, we have chosen the following parameter set, that is listed in Table 1 below.

Param	Value	Param	Value	Param	Value
δ_j	2	μ_j	0.5	s	0.1
δ_0	0.8	μ_0	0.1	Ly	100
ς_j	0.9	α	0.4	R	5
η_j	1	β	0.5	K_0	0.2

Table 1. Base run parameter set

In the table above, the parameters with an index *0* refer to the basic R&D process, while those indexed with a *j* are GPT specific. The actual value for each GPT is drawn from a uniform distribution on the range from *0* to the corresponding value in the table above. The expected value of each parameter that is GPT specific is therefore 50 percent of the values in the Table. An exception is the GPT expansion scope parameter ς_j. The value in the table represents the minimum value of that parameter, whereas the actual value is drawn randomly from the uniform distribution ranging from the minimum to a value equal to 1. Finally, the distribution parameters $c_{0,j}$ are drawn from a uniform distribution again, where the expected value is 2.5 percent above the value realized for the previous GPT, where we do allow for a fall in the value of the distribution parameter with uniform probability 0.25.[19] The distribution parameter of the core of the first GPT is drawn from a uniform random distribution on the range 0-1. The random parameters are summarized in Figure 1 below.

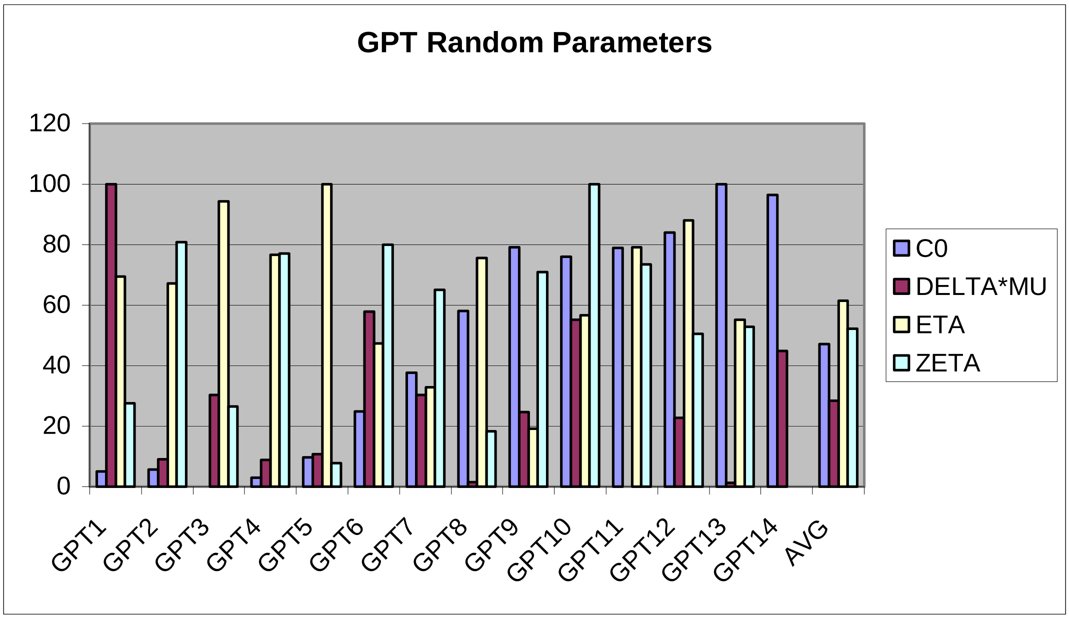

Figure 1. GPT characteristics

In Figure 1, the bars labelled 'AVG' correspond to the average value of the parameters over all GPT's 1-14, since those are the GPT's that arrived during a simulation period ranging from 1-300. The characteristics of these GPT's are summarised in the columns labelled GPT1-GPT14. Since the effective productivity of the corresponding applied R&D processes

[19] This is implemented by means of the following rule x=x0.(1+4*r*y-y). Here y is the required average percentage increase in x w.r.t. x0, while r is a uniform random variable on the range 0-1. Since the expected value for r is equal to 0.5, we will have that the expected value for x is equal to x0.(1+y), as required. If however, r is below 0.25, x<x0, so x can indeed fall, even when the drift is equal to y>0.

are given by the products of δ_j and μ_j, we do not present these parameters separately, although they are conceptually different. The vertical axis of the Figure measures the relative distance of a parameter value in the range of all values of that parameter for all GPT's 1-14. The respective ranges are listed in Table 2 below.

	C0	DELTA*MU	ETA	ZETA
MINOBS	0.000330	0.002940	0.028786	0.902272
MAXOBS	0.000465	0.814718	0.954599	0.988458
RANGE	0.000135	0.81177	0.925813	0.086186

Table 2. Ranges of variation for the random parameters

In Table 2, MINOBS is the minimum value of the parameter in question and MAXOBS is the corresponding maximum value. RANGE is the difference between MAXOBS and MINOBS. The entries in Figure 1 can now readily be obtained from the actual random parameter value and the entries in Table 2 above.[20]

Looking at Figure 1, we can observe two broad tendencies: C0 is indeed rising on average with the GPT index, as it should be, while there is no definite patterns in the other parameters, again as it should be.

4.3 Simulation Results and Interpretation

Based on Figure 1, we can categorise the various GPT's by their above or below average (positive) impact on profitability. So, since η influences profitability in a negative way, a value below average for η means an above average positive impact on profitability. Henceforth, we obtain the following table, where a + means above average impact on profitability, a minus means below average impact on profitability and a zero means roughly average impact on profitability.

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[20] This can be done by applying the following 'rule': entry Table 1 = 100*(actual value random parameter – corresponding MINOBS)/corresponding RANGE.

GPT	$C_{0,j}$	$\mu_j \cdot \delta_j$	ς_j	η_j	Arrival Period	Category
1	-	++	-	-	1	Success
2	-	-	+	-	11	Niche Success
3	-	0	-	--	52	Niche success
4	-	-	+	-	54	Failure
5	-	-	-	--	56	Failure
6	-	+	+	+	57	Big Hit
7	0	0	+	+	119	Success
8	0	--	-	-	159	Failure
9	+	0	+	+	163	Success
10	+	+	+	+	182	Success
11	+	--	+	-	217	Failure
12	+	0	0	-	237	Niche success
13	+	--	0	+	245	Failure
14	+	+	-	++	280	Big Hit?

Table 3. A tentative GPT categorization

The categorization in the last column comes from the development over time of the number of peripherals. The latter is shown in Figure 3. But first we present the development over time of the number of GPT's that are active at each moment in time in Figure 2, for the parameter sets provided in Tables 1 and 2.

The economy starts with only one active GPT and develops more potential GPT's so that after 300 periods (further called years for ease of exposition) it has 14 potential GPT's available. A new GPT arrives on average every 21 periods, but the intervals between the arrival of one GPT and the next are far from regular. In this simulation run, for instance, the 'Fifties' are a very innovative period, witnessing the arrival of three new GPT's, but after this innovative burst it takes more than 60 years for GPT 7 to arrive.

In Figure 3, we present the development over time of the number of peripherals by GPT. This figure makes it clear that the success of a *potential* GPT depends not only on its intrinsic characteristics, but also on the timing of its arrival. GPT 1, for instance, develops a large number of peripherals, but only because it faces no serious competition until GPT 3 arrives. After the arrival of GPT 3 the growth of GPT 1 slows down dramatically. It is quite evident that GPT 1 would have been much less successful if GPT 3 had arrived earlier. GPT 3, in turn, might have become a real success had it not been outcompeted by the superior GPT 6. This is why we have categorised GPT 3 as a “niche success” in Table 3.

Measuring the GPT character by the sheer extension of each GPT in terms of the number of peripherals, GPT 6 is clearly the “big hit”. GPT’s 1, 7, 9, and in particular 10 also do reasonably well, earning a slot in the category “success”. GPT 12 has developed only 7 peripherals by the end of the simulation period, which is not much, but this number is still growing, so we generously put it in the category “niche success”. At the end of the period, we see GPT 14 emerging in the year 280 and developing 9 peripherals in only 19 years. If its growth continues at such a pace, it may become a big hit in the future. Finally, turning to the bottom of the graph, we see that a number of potential GPT’s have never developed more than a handful of peripherals. These are ‘failed’ GPTs’, consisting only of a core or of a core and very few peripherals. Such technologies – while possibly quite profitable – certainly do not fulfil the definition of a GPT. The interesting point about these ‘failed’ GPT’s is that we see them as non-GPT’s in our ex post perspective, but the researchers who developed them were seeing them as potential GPT’s, because they had no way of knowing their characteristics ex ante.

The number of peripherals allows us to say something about the usefulness of a potential GPT, but it does not tell us very much about the economic impact of the GPT. But one important characteristic of a GPT is that it affects a large share of the economy. In order to measure the economic impact of a GPT - its “market share” so to speak – we show the Eulerian contribution of each GPT to effective capital K_e.[21] in Figure 4.

The curves in Figure 4 can also be thought of as “diffusion curves”, because they show a similar phenomenon.[22] These diffusion curves show a cyclical pattern, which is consistent with some long wave views on economic development. Freeman and Perez (1988), for example, identify five Kondratieffs in economic history since the late 18th century. Such Kondratieffs are characterised by the dominant GPT of their times, for instance the “steam

[21] Since K_e is a linear homogenous function of the individual $K_{e,j}$’s, we have the Euler equation:

$$K_e = \sum_{i=1}^{A} \frac{\partial K_e}{\partial K_{e,j}} \cdot K_{e,j} \;\; => \;\; 1 = \sum_{i=1}^{A} \frac{\partial K_e}{\partial K_{e,j}} \cdot K_{e,j} / K_e \equiv \sum_{j=1}^{A} \theta_j$$

where θ_j therefore measures the contribution according to Euler of effective capital belonging to GPT j to aggregate effective capital.

[22] The reader may notice that in the diffusion literature the usual form of a diffusion curve is S-shaped, in contrast to our “diffusion curves”. The reason for the common S-shape are learning effects, scale economies, imperfect information etc. In our model, however, such effects are absent, so that “diffusion” starts at a fast pace, slowing down as the extension possibilities (indicated by ζ_j) of the GPT are being exploited.

power and railway Kondratieff" in the mid-19th century or the "information and communication Kondratieff" that started in the late 20th century.

In our model we can identify similar Kondratieffs: The "GPT 1 Kondratieff" gives way to the "GPT 6 Kondratieff" around the year 60, when GPT 6 overtakes GPT 1 in Figure 4. The GPT 6 Kondratieff reaches its climax between 110 and 120. It then undergoes a transformation into the "GPT 6 and 7 Kondratieff", during which GPT 6 remains the dominant GPT, but GPT 7 also plays a major role. In the year 159 (when GPT 9 is invented), the simulation economy begins a long transition towards the "GPT 9 Kondratieff", which begins around the year 200 and finally gives way to the "GPT 10 Kondratieff" around 245. Towards the end of the period, another major transition comes about: GPT 14 starts its steep rise and becomes the dominant GPT in terms of its contribution to K_e only 12 years after being invented.

Note that in contrast to many long wave theories, there is nothing mechanistic in the coming and going of Kondratieffs in our model. Due to the structure of the model every GPT will at some time run out of further extension possibilities, and the search for a new GPT begins, but the length of these "long waves" is endogenously determined and highly variable.

Another interesting observation concerns the length of the transition periods between two successive Kondratieffs. The transition from the GPT 1 Kondratieff to the GPT 6 Kondratieff is rather quick, but others are not. GPT 10, for example, is invented in 182 (when GPT 6 is still dominant), and it takes more than 60 years for GPT 10 to assume a dominant position. In order to explain the speed of these transitions, it is quite useful to look at the developments in the R&D sector.

Figure 5 depicts the allocation of researchers to applied R&D on the available GPTs. This is where things get exciting, because the R&D sector is really what drives the economy in our model, even though the fixed number of researchers (5) is quite low in comparison to the number of final output workers (100). In order to gain a complete overview over the R&D sector, we will also refer to Figure 6, which depicts the allocation of researchers between basic R&D and applied R&D.

Just like in Figure 4, we clearly observe cycles in R&D. The usual pattern is as follows. A new, potential GPT enters the market. If it is an attractive technology for research (in terms of the characteristics shown in Table 1), research labour moves into applied R&D on the new technology. This can produce drastic changes: The really successful GPT's attract around 90% of all researchers, and GPT 14 is so attractive that it absorbs virtually all research labour. Naturally, this crowds out the other GPT's as well as basic research. The jumps that we observe in Figure 6 all relate to the introduction of a new, successful GPT. We can also see from these two figures that the arrival of 'failed' GPT's, such as that of GPT 11 in the year 217, does not affect the allocation of researchers, because 'failed' GPT's do not attract any applied R&D.

Upon introduction of an attractive GPT, a lot of researchers move into applied R&D on that GPT. They quickly develop a number of peripherals. Due to our assumptions about the nature of technologies, however, they run out of extension possibilities, and further research on this GPT becomes less attractive. Thus, researchers move gradually into applied R&D on other GPT's and into basic R&D on the next potential GPT. One can nicely see this at the end of the period: GPT 14 attracts a lot of researchers upon introduction, but its scope for extension (ζ_{14}) is low. Therefore, the attractiveness of applied R&D diminishes quickly, and GPT 10 is starting to experience a renaissance at least in terms of R&D.

Generally, we observe that the R&D sector is "leading" the rest of the economy. When a new GPT is introduced, the R&D sector moves in quickly, developing products and getting hold of the patents. It then moves into other activities, either basic R&D or applied R&D on the next GPT. This phenomenon is especially obvious in the case of GPT's 9 and 10. The "GPT 9 Kondratieff" begins in the year 200, although GPT 9 was invented nearly 40 years before that. In the meantime, research labour has to a large extent left the GPT 9 sector and has moved on to applied R&D on GPT 10.

It is precisely this movement between different R&D activities that generate growth in the model. Each individual GPT's scope for extension is limited, and when this limit is approached, researchers move towards basic R&D, searching for the next GPT. These movements generate cycles in the composition of the economy and its growth rate. This last claim can be seen in Figure 7, which shows the growth rate of output and a dummy that takes the value 1 in those years when a new GPT arrives. Successful GPT's have two effects on growth: First, they raise output immediately in the period after their introduction, simply because of our love of variety assumption. But secondly, and more importantly, they raise the average growth rate over a period of several decades, as researchers are exploiting the new possibilities for applied R&D. This can be seen in the case of GPT 6, for example. Right after its arrival, the growth rate jumps up to an impressive 6.7 percent. Then, during the "GPT 6 Kondratieff", we observe growth rates that are (on average) gradually declining. It is the arrival of GPT 7 in the year 119 that sets off another period of high growth rates.

The growth impact of new GPT's, of course, depends on the GPT's pervasiveness. In the extreme case, a total failure with no peripherals at all, we have only a short-lived growth hike due to the love of variety effect. This uncertainty about future growth rates is also a realistic feature of the model: We know that a new successful GPT will arrive, and that it will set off a period of high growth, but we can never know when this will happen.

4.4 Policy Relevance

Clearly, in our model the transitions from one technological regime to another are not painful because many consequences of real world transition are missing: creative destruction in the form of obsolescence of skills and capital, unemployment, redistribution of income and wealth etc. In order to alleviate the pain associated with such adjustments, some government intervention may be called for that to some extent mitigates the fierceness of the reallocations of labour implied by the arrival and subsequent takeover of new GPTs. Extensions of the model in this direction are left for future research, though.

The model simulations we have performed clearly illustrate that the engine of growth in this model is a two-stroke engine. The two cylinders, basic R&D and applied R&D, fire in turns, the first churning out a new GPT and the latter sputtering out a number of peripherals for the new GPT. The R&D market performs the function of an ignition distributor: Whenever the applied R&D cylinder loses momentum, the market reallocates researchers to the basic R&D cylinder in order to produce a new GPT. Note that this mechanism is not necessarily efficient. There are all sorts of market imperfections involved, and there may be ample scope for government policy to tune up the engine by influencing the allocation of R&D efforts. Such policy interventions may result in higher acceleration (faster growth) or more horsepower (higher consumption per capita). Again, this is left for future research.

5. Summary and Conclusion

In this paper we have presented a simple model that allows for the occurrence of innovations that can develop into GPTs. The model consists of a multi-layered Ethier production function and has roughly the same structure as Romer (1990) and Aghion and Howitt (1992). It is supposed to explain the cyclical pattern in output growth, without having to resort to contractions of final output production itself in response to favourable R&D prospects, since the mobility of labour between final output production and R&D activities seems to be limited to say the least, quite apart from the low share of R&D workers in total employment, and therefore the limited effect of their complete reallocation towards final output production. In our model, R&D workers can only switch between different types of R&D: basic R&D that injects the core of a potentially new GPT into the economic system, and that supplies the potential for further expansion through applied R&D that adds peripherals to this core. We show using simulations with this model, in which the arrival of the basic and applied innovations are governed by Poisson processes as in Aghion and Howitt (1992) that the arrival of a successful GPT does indeed bring about a reallocation of R&D activities towards applied R&D, thus (on average) postponing the moment of arrival of the next GPT. Meanwhile, applied R&D raises the productivity of the GPT as a whole, but the profitability of finding the next peripheral falls in the process, thus diminishing the incentives to engage in further applied R&D and increasing the incentives to move into basic R&D activities again. Thus we obtained a cyclical pattern in growth that is partly driven by the arrival of the underlying GPTs but that are also internally driven by the continuing development of and development races between active GPTs in the absence of the arrival of new ones. We even have seen instances of failed GPTs that are a contradictio in terminis, of course. However, by generating failed GPTs, the model illustrates our notion that a GPT is essentially an ex post concept, that is therefore relatively empty, unless one is able to link the ex post performance of an innovation to its basic characteristics in terms of wider applicability, scope for expansion, efficiency of expansion processes as well as associated production processes, and so on. We have shown that exactly these characteristics do indeed define the success or failure of a GPT, i.e. the answer to the question whether to be or not to be a real GPT.

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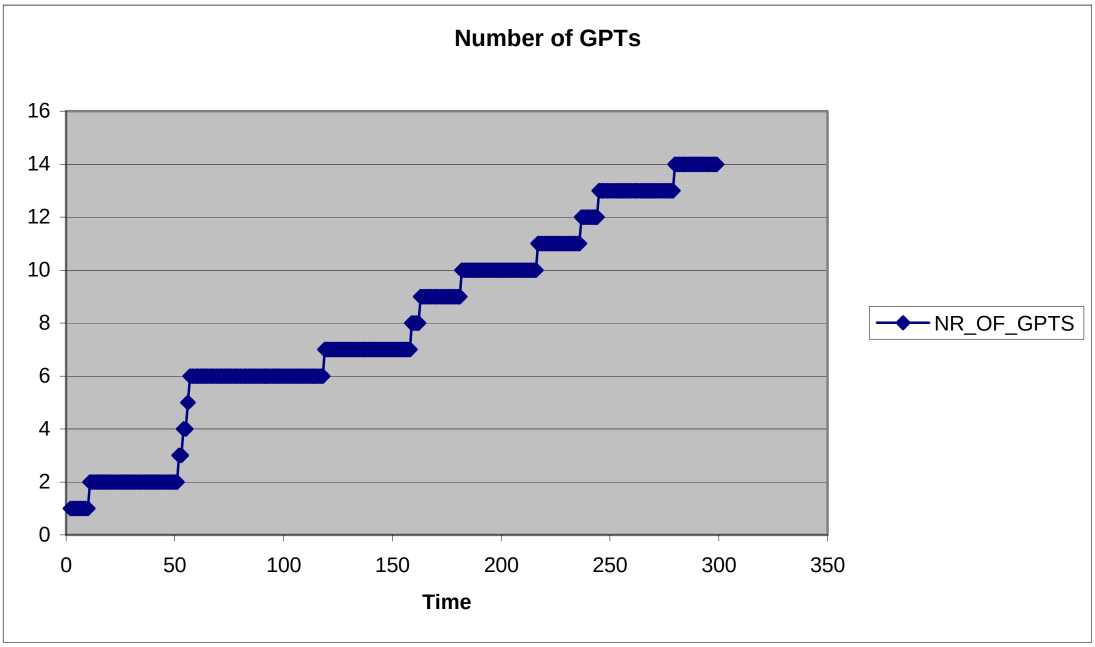

Figure 2:: The number of GPTs over time

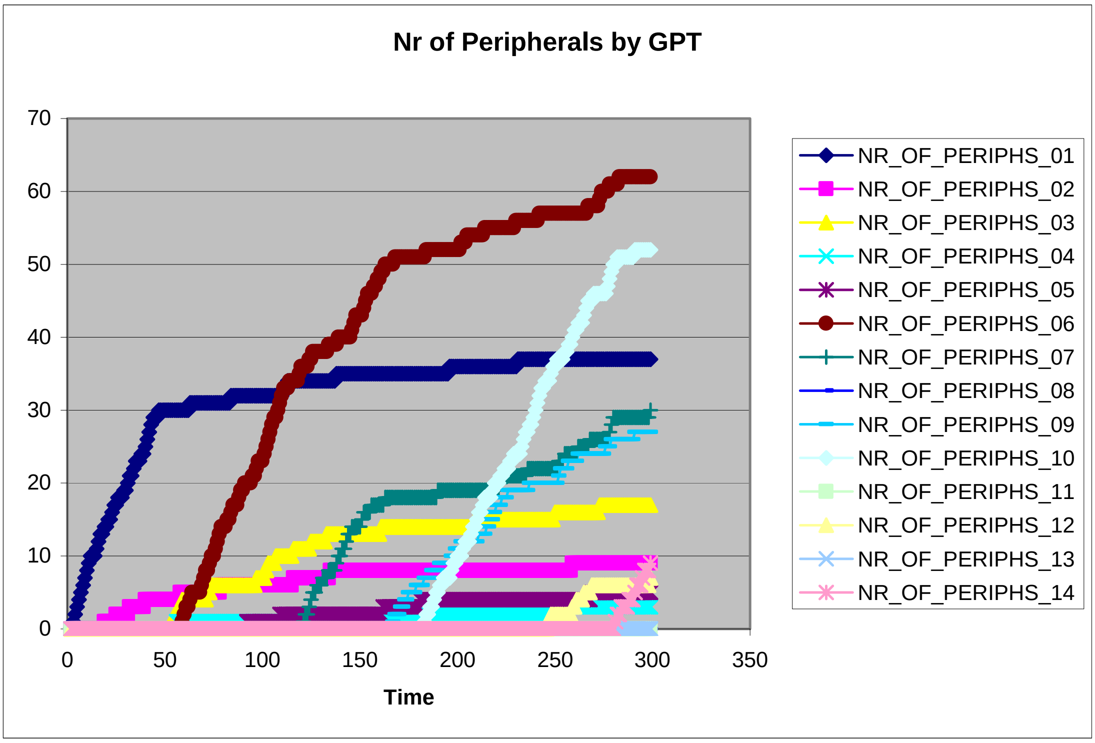

Figure 3: The number of peripherals for each GPT

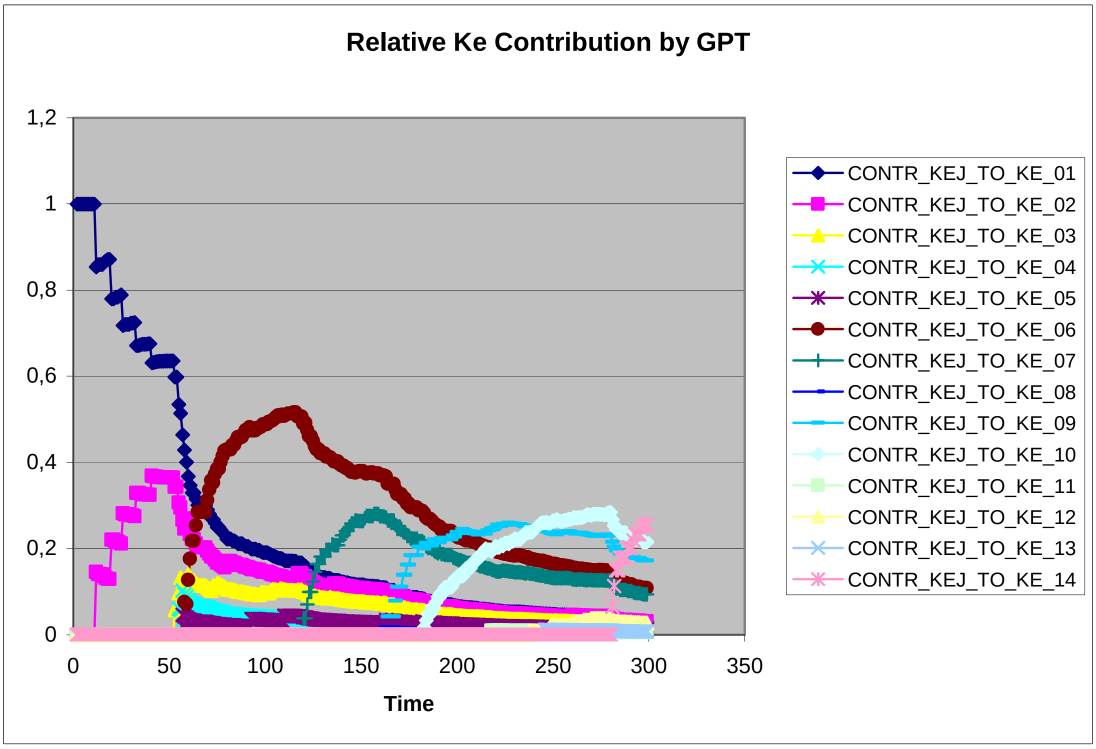

Figure 4: Relative contribution to effective capital

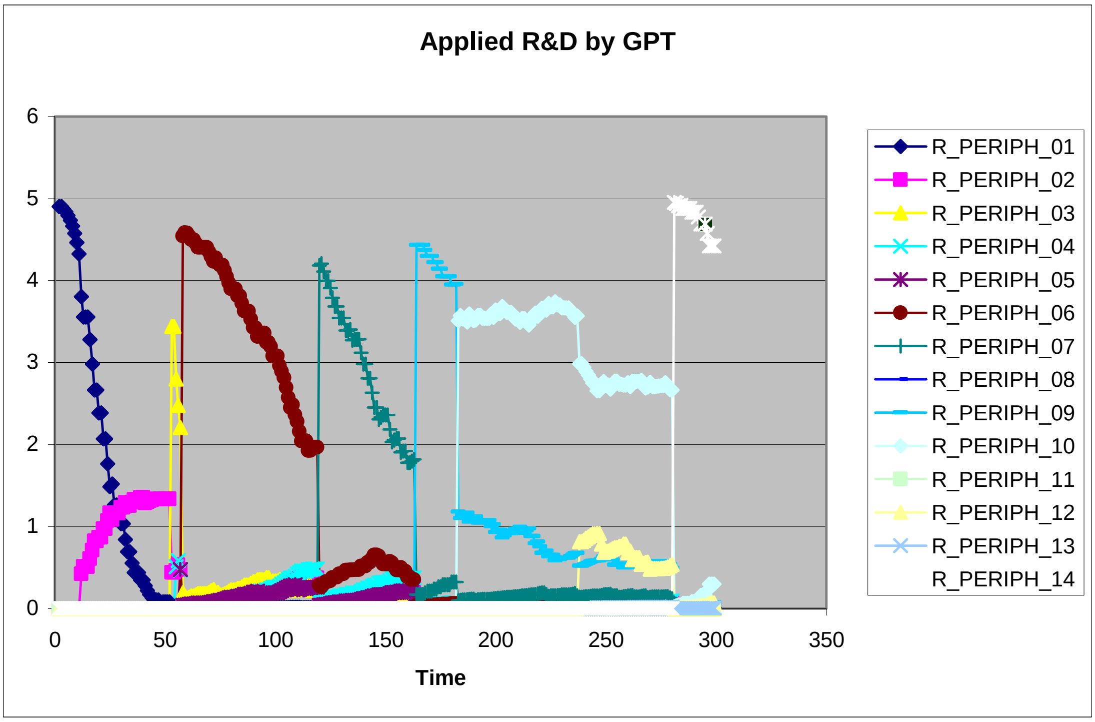

Figure 5: Applied R&D trade-offs

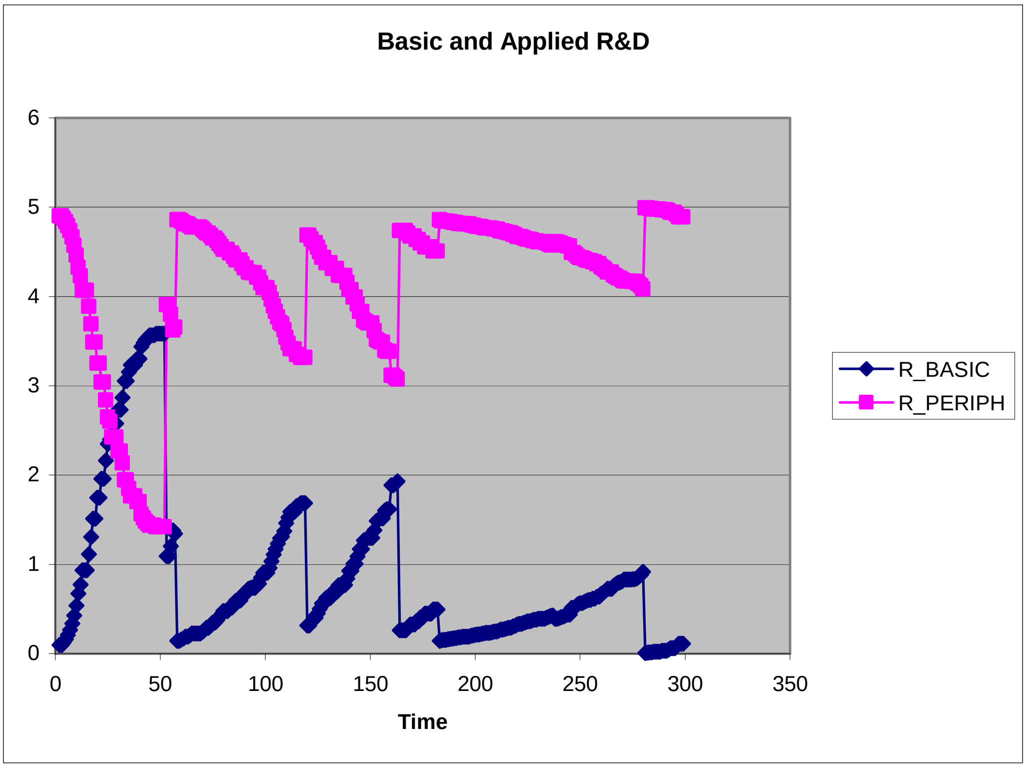

Figure 6: Basic vs. applied R&D

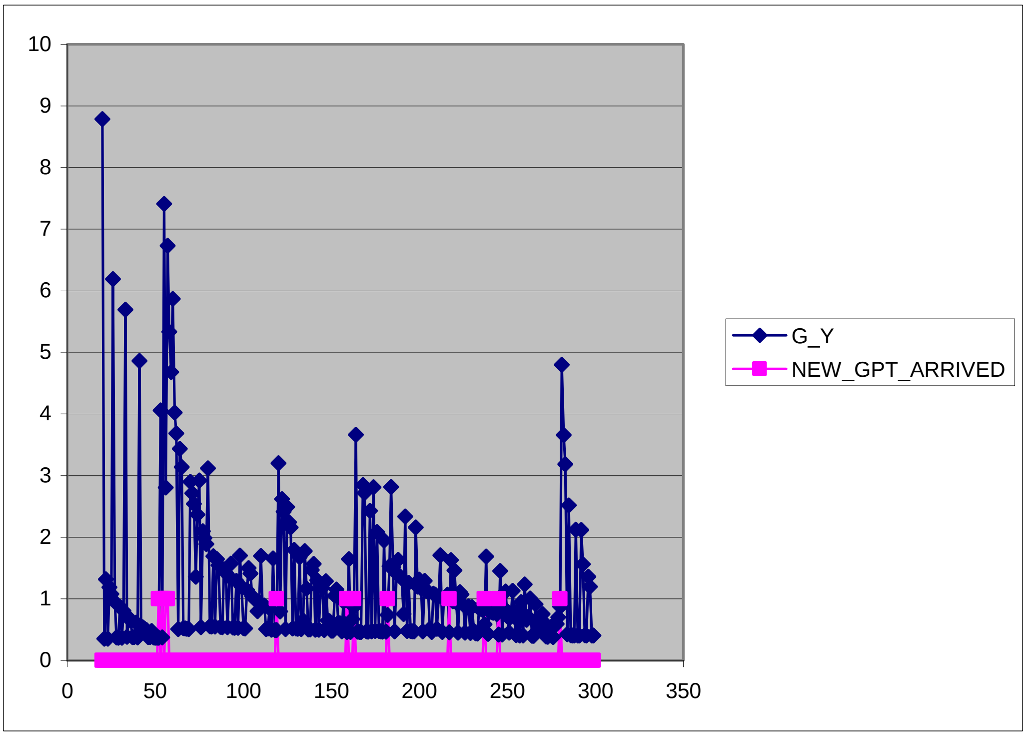

Figure 7: Final output growth