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Malthus irrelevant?

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Abstract. We estimate and make population forecasts with Foley's (2000) model in three different ways. The population forecasts for high, middle and low-income countries and the world aggregate are quite good and suggest that the omitted variable bias from its simplicity may be small. Estimation of the model as a system shows that Malthusian behaviour - defined as increasing population growth through increasing per capita income - cannot be found for any of the income groups of the World Bank classification nor for Sub-Saharan Africa or the world as a whole, and also not for countries with per capita income below \$1200 in a panel estimate. This confirms what Foley found for world aggregate data. For world aggregate data and for the low-income countries we find increasing returns to scale – as Foley did for world aggregate data - , but for the other groups decreasing returns (outweighed by a positive time trend except for Sub-Saharan Africa and the u1200 group). For the panel of countries with income below \$1200, per capita income is stagnant for the period 1970-2002 in spite of the positive growth rates of the period 1991-2002. In the estimate of the panel for Foley's model, the time trend in GDP per capita growth is as strong as the population growth in connection with decreasing returns to scale. Together with the absence of Malthusian behaviour this seems to suggest a strong role for the population growth problem as seen by David Ricardo.

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1 Introduction

There is a widespread view that history was not friendly to the ideas of Thomas Malthus (see Foley 2000). According to this view his ideas were relevant from the year 1000 until about 1800 but not thereafter (see van den Berg 2001). A closer look at the argument in both cases shows that the judgement is based on the use of data on the world level showing that there are positive growth rates in spite of population growth. However, the growth of per capita income may occur in the rich part of the world whereas the population growth may occur in the poorer part of the world. In this paper we want to test whether or not this judgement may turn out to be premature when we look at more disaggregated data and in particular those of low-income countries. In section 2 we report Foley's model and derive some (formally) special cases, which may be relevant for the world as a whole or for certain income groups. In section 3 we provide a single equation estimate and population forecasts based on Foley's model, which show quite realistic variants of the model in the sense that the model produces reasonable forecasts. In section 4 the model is estimated in its system form in order to get the information in regard to returns to scale and Malthusian behaviour, where the latter is defined as higher per capita income leading to higher population growth. In section 5 we turn to van

den Berg's argument and show that for three different levels of income in terms of constant 1995 US dollars, two of which are estimates for income in Europe as of 1800, the percentage of countries below those values has not decreased since 1980. They are about stagnant in income per capita. Population growth may be one of the reasons. Moreover, the kernel density distribution for GDP per capita shows one peak at an almost constant level of per capita income of 1000 US dollar (1995) since 1960 and an increasing dispersion for countries with higher per capita income. These data all in all suggests that the group of 71 countries below \$1200 (1995 dollars) was about stagnant in the period 1970-2002 (when 1960 is included the growth is still below a half percent) in spite of the positive growth rate for the period 1991-2002. Also for this group of countries we confirm Foley's result of absence of Malthusian behaviour with decreasing returns of GDP per capita to population growth though. Section 6 summarizes and concludes that the population growth problem is still there in the way Ricardo saw it but Malthusian behaviour is not.

FIGURE 1 OVER HERE

2 Foley's model and some special cases

Foley's model consists of two curves with inverted U-shape (see Figure 1). Income per capita is a function of population with an inverted u-shape in the lower graph of figure 1, which we call the production function. In the upward sloping (Smithian) part we have increasing returns to population growth and in the downward sloping (Malthusian) part decreasing returns to population or labour. Changes in the population are an inverted u-shape function of income per capita in the upper part of figure 1. In the upward sloping, Malthusian part population varies positively with income, and in the downward sloping Smithian part population varies negatively with income. We call it the population growth curve.

Together they work as follows. Select a value of the population. Then you find income per capita on the production function. With that income-per-capita value you go to the upper part and find the change in population, which tells you in which direction to go from your initial value. At a fertility rate of $f = 2$ population is stationary by assumption. The figure, as drawn by Foley, has four steady states. If you choose one steady-state value, imagine you are a bit to the right of it, then population is larger and output per capita is larger if you are in the increasing returns part and lower if you are in the decreasing returns part. In the upper panel we have the change in the population, which is positive if $f > 2$ ($dN > 0$) or negative if $f < 2$ ($dN < 0$). If population increases, the steady state from which you started is unstable and if it decreases it is stable. The steady states are characterized by indices M and S for ‘Malthusian’ and ‘Smithian’ as well as U and S for ‘unstable’ and ‘stable’. Foley looks at world data, which can be plotted in the part with increasing returns and falling fertility. The steady state, which is relevant for world data then is a stable Smithian one. Once the stable Smithian steady state is reached, GDP per capita stops growing and there is a constant population. This view resembles a class of growth models sometimes called ‘semi-endogenous’ where the long run growth rate goes to zero as the population growth rate goes to zero as in Arrow (1962). The steady state in the decreasing returns part and the upward sloping fertility part is a stable Malthusian one. On the very left we have a Malthusian one, which is unstable because of the increasing returns to scale. The second Smithian steady state in the falling part of the fertility curve is unstable because of the decreasing returns to scale. Now we derive some special cases for a given population growth curve:

Case 1: Productivity may be sufficiently low to let the Smithian steady states vanish.

Case 2: In this case there exists no decreasing-returns-to-scale part. This assumption is plausible for the rich countries and for the world (as in Foley’s Loess fit regression), which are to some extent dominated by the rich countries, because the world as a whole has no signs

of decreasing returns so far. Then the Malthusian, unstable steady state and the stable Smithian one are still there. The other steady states are absent if the production function has no decreasing-returns-to-scale.

Case 3: The increasing returns part is not steep and the production function has per capita output larger than zero at $N = 0$. If this value of per capita income is high enough, the Malthusian unstable steady state vanishes. If, in addition, there is no decreasing-returns part, there is only one stable (Smithian) steady state.

Case 4: The production function has no increasing-returns-to-scale part. This assumption is a priori plausible for stagnant low-income and lower middle income countries. In this case the unstable, Smithian steady state and the stable Malthusian remain in the graph of figure 1. The two steady states more to the left vanish.

The two graphs in Figure 1 can be formalized as two linear-quadratic functions. Note that $f-2$ in Foley's model is not a percentage rate, but rather the change in population. Therefore we write the population growth curve – denoting dN as the change in population, y as per capita income unless it refers directly to figure 1 where it is denoted as x - as follows:

$$dN = a + by - cy^2 \quad (1)$$

The production function is (according to the lower panel of Figure 1)

$$y = e + fN - gN^2 \quad (2)$$

Here, a , b , c , e , f and g , are simple coefficients with no other meaning. To both equations a time trend for historical shifts and a residual for econometric purposes can be added. Insertion of equation (2) into (1) yields a polynomial of the fourth degree:

$$dN = a + b(e + fN - gN^2) - c(e + fN - gN^2)^2 = \beta_0 + \beta_1 N + \beta_2 N^2 + \beta_3 N^3 + \beta_4 N^4 \quad (3)$$

In principle such a polynomial can have four stationary values as Foley's model has. Estimation of the function would reveal whether or not this is the case. This follows from getting signs for coefficients and plotting the estimated function.

In case 1, parameters e and f must be low. In case 2, c is zero or at least very low such that the linear part is dominating. In case 3, e must be sufficiently large to guarantee that $y(N=0) > y(dN=0)$.

Obviously, the special case of equation (3) of having only the linear part is Kremer's (1993) model and the linear-quadratic part is the well-known Verhulst-Pearl equation (see Gandolfo 1996). Foley's model extends these cases by the terms with exponents three and four.

This is a very simple economic model. It has no physical or human capital or technical progress. In econometric terms we would therefore expect to have an omitted variable bias. A justification for the simplicity could be that it is producing good forecasts. These might indicate that the bias is small. Therefore we first focus on the forecasts.

Among the three classes of methods for the purpose of making population forecasts - forecasting time trends of level or growth rate of population, historical analogy, and social and economic theory (Lee 1991) - the use of equations like (3) or special cases of it is a combination of using economic theory and forecasting.

TABLE 1 OVER HERE

3 Population forecasting with Foley's model

We estimate equation (3) in the simple OLS mode using data for population per capita income (in constant 1995 \$) from the World Development Indicators. It is a differential equation and

as such analysis of its stability is similar to a unit root test. We do not have the corresponding critical values for the analysis of Foley's model though.² In order to correct for autocorrelation we have added lagged dependent variables but they were mostly insignificant. In order to correct for endogeneity we have tried lagged right-hand side variables as instruments added to the other regressors in the GMM mode but the results for the forecasts are worse than those used below. Following Giersbergen and de Beer (1997) it is assumed that there is a lag of two years before population change, dN , reacts to changes in income, y . The results of our estimates are summarized in Table 1. In these estimates all exponential terms are significant at least at the 10% level and so is a squared time trend. However, when trying to run forecasts with these estimates it turns out that the difference equation is unstable, either because of the third (for high-income countries) or fourth exponent or because of the quadratic time trend. When dropping the terms causing instability, we get the regressions in Table 2. These estimates are still quite good in terms of significance, adjusted R-squared, absence of autocorrelation and heteroscedasticity and the Ramsey RESET test. The major disadvantage compared to the estimates in Table 1 is the increased number of observations indicating parameter non-constancy according to the one-step-forecast test and the n-step forecast test. The CUSUM of squares test is also worse. The forecasting results from these estimates are presented in Table 3 and in Figures B12-B6 (see appendix in the working paper version). The prediction for the world as a whole is 8.6 billion people in 2100. For high-income countries the prediction is 1.65 billion, for upper-middle income countries .78 billion, for lower-middle-income countries (including China) 2.76 billion and for lower income countries (including India) 5.3 or 4.3 for two different estimates (see Figures A6 and A7). These disaggregated predictions sum up to 10.5 or 9.5 billion people, which is slightly higher

² Similar models are used for the analysis of diffusion processes for new goods. The differential equations used there differ from those in the standard unit root tests. Unit roots go unmentioned (see Putsis and Srinivasan (2000)). The justification for this may be that these models imply a different assumption on the true model generating the variable. For these models critical values are available only in exceptional cases (see Lucke and Lüthkepool 2004).

than for world aggregate data. The forecast statistics, such as the Theil coefficient, indicate that the forecasts are reasonably good. In particular, the covariance proportion of the mean squared error is in all case above .99 with a correspondingly low bias proportion and variance proportion. The number of observations indicating doubts on parameter constancy is a weak point. However, rather than complicating the model or the estimation method, it may be better to repeat the forecast every year. Given its simplicity, this requires very little effort.

TABLE 2 OVER HERE

Our estimates are within the range of those obtained with other methods (see Table 4 in billion). This shows that Foley's model is useful for forecasts. In order to carry out the analysis in regard to the relevance of Malthus' ideas, we will plot next the graphs of the estimated equations and relate them to the special cases discussed above.

TABLE 4 OVER HERE

Figure A2 shows the plot of the regression used for population forecasting for the *world aggregate* in periods $t = 0, 10, 100, 140$ corresponding to the years 1960, 1970, 2060 and 2100. There are only two shifting or quasi steady states for given values of time. In spite of the time shift we will speak for simplicity of steady states. The one at a lower population is unstable and the other is stable. Over time the curve shifts upward. This might be due to technical progress, factor accumulation or other effects captured by the time trend. As a consequence the unstable steady state vanishes. Thus, here Foley is supported under the additional assumption of increasing returns: for the world as a whole history has no support for the ideas of Thomas Malthus, because the unstable steady state vanishes over time. The

actual and forecasted population data of 3.06, 3.68, 8.03, 8.59 billion people for 1960, 1970, 2060 and 2100 indicate that the population is very close to the moving steady state. However, the model has an unstable steady state to the left of the stable steady state. Without additional information on returns to scale we cannot know whether or not the stable one is the Smithian steady state.

The population growth curve for the *high-income countries* in Figure A3 looks very similar. The population numbers, H, for the years 1960, 1970, 2060 and 2100 are .6772, .758, 1.37, 1.65 billion. In later years these numbers are fairly close to the moving steady state, but not in earlier years. One can take it from the World Development Indicators that (not shown) the fertility rate for these countries has fallen below the value of two in the beginning of the seventies. Not being in the steady state is probably due to immigration as far as population is concerned and to technical progress in regard to GDP per capita. Again information in regard to scale economies is needed in order to decide whether the stable steady state is Malthusian or Smithian.

For the *upper-middle income countries* we find only one steady state shown in Figure A4. Depending on the returns to population we may either have a stable but shifting Malthusian or Smithian one. The population, U, in 1960, 1970, 2060 and 2100 is .236, .294, .703, and .779 billion.

For *lower middle-income countries* we find two steady states again in Figure A5. The unstable steady state is vanishing after 2100 as the curve moves up over time. Population growth is quite high. China is part of this group. The population, M, in 1960, 1970, 2060 and 2100 is 1.1, 1.35, 2.65, 2.76 billion, which is getting closer to the moving equilibrium.

For *low-income countries* there is only one moving steady state in Figure A6. The population, L, in 1960, 1970, 2060 and 2100 is 1.01, 1.27, 4.4, 5.31 billion. These values are very close to the steady state. The shifting stable steady state is either a Malthusian one or a

Smithian one, again depending on the returns to scale. The graph for the second scenario of Table 2 is not plotted. Its time shift is less strong but the curvature is very similar.

So far this section has shown that Foley's model delivers reasonable forecasts and therefore may be useful in spite of the missing variables like physical and human capital. Next, we will estimate the model in its form with two equations in order to get the information on returns to scale. This will allow us to see whether or not Malthusian steady states and behaviour are irrelevant.

4 A system estimation of Foley's Model

In this section we estimate equations (1) and (2) as a system. As all variables have unit roots and we assume that they are cointegrated³ we take first differences of the two equations and insert the lags of the level equations (1) and (2) for the lagged residual term and multiply it with a coefficient. This yields the following variant of an error-correction model.

$$d(dN) = c(9)*[d(N(-1)) - c(1) - c(2)*y(-3) - c(3)*(y(-3))^2 - c(4)*t] + c(2)*d(y(-2)) + c(3)*d(y(-2)^2) + c(4) + u(t)$$

$$d(y) = c(10)*[y(-1) - c(5) - c(6)*N(-1) - c(7)*(N(-1))^2 - c(8)*t] + c(6)*d(N) + c(7)*d(N)^2 + c(8) + v(t)$$

The $c(i)$ are the regression coefficients, and '*' indicates a multiplication. The added residuals are $u(t)$ and $v(t)$. As all right-hand side variables are endogenous in the sense that a certain value of the residuals has an impact on the left-hand side variable and therefore on future values of the right-hand side variables, we use lagged regressors as instruments in a general method of moments (GMM) estimation. As the right-hand side has levels as well as first differences of some of the regressors we have to take care of interdependences among them in order to avoid linear dependence among instruments. We can use either the levels or the first

³ As pointed out in the previous section we do not have the critical values for unit root and cointegration tests that would correspond to the Foley model.

differences as instruments. We tried both and the results were almost identical. The estimation results are summarized in Tables 4 and 5 and are plotted in Figures A13-A17.

TABLE 5 OVER HERE

For aggregate *world* data we find the result in column one of Table 5 and Figure A7. Time trends are highly insignificant and therefore have been dropped. As we will see below, the reason for this is that time trends of the population curve go into opposite directions in the groups with the different income levels and are positive for all productions functions but those of the low-income countries. The richer the countries according to the income classification the stronger the effect of the time trend in the production function (see Tables 4 and 5, coefficient $c(8)$). The population growth curve for the world aggregate has the expected inverted u-shape. The production function has only the increasing returns part. By implication, there is an unstable Malthusian steady state and a stable Smithian one as Foley found it in his analysis. As the world is at higher population and income levels than the unstable steady state – in 2000 income per capita is 5640 constant 1995 US\$ and the population is above 6 billion – population would continue to grow until income is about \$9000 and population to about 10 billion. These values are at the higher end of the population prediction of the previous section. The value of \$5640 is on the falling part of the population growth curve and therefore there is no Malthusian behaviour – increasing population change as income is higher - on the world level anymore. This is about what Foley found using Loess fit regressions. Now we will check the results on the more disaggregated level of income groups.

In Figure A8 the result for the *high income* countries is plotted from the estimate of column two of Table 5 for the years 1960, 2000, 2060, 2100. The population change is

increasing in income, but the time trend is pulling the curve down. This result is in accordance with recent literature arguing that it is not the income that reduces population growth but rather human capital formation and change in gender issues as captured by the time trend here. Children have positive income elasticity. The production function has a u-shaped form, but population is always to the left of the minimum of the curve: for the year 2000 we have $N = .975$ billion, $y = 29200$, $dN > 0$. By implication, we are in the Malthusian part of the fertility curve and the decreasing returns part of the production function. There are only Malthusian steady states, which are shifted over time. The relevant one to the left of the minimum of the production function is stable. However, the time trend shifts the population change curve down and the production function up and in 1960 there is not yet a steady state and in 2060 there is no one anymore because the income of about \$40000 at which dN would be zero is not generated by any population level but rather is always higher. High-income countries therefore exhibit the classical constellation of time, or technical change and factor accumulation, versus decreasing returns and population growth, though under conditions of human capital formation. The time trend is strongly dominating the data. We feel that it is an open issue here whether or not the upward sloping part of the population growth curve has to be interpreted in the Malthusian spirit. It is equally plausible to argue that children are like a luxury good. This then should not be interpreted as Malthusian behaviour, in particular because the phenomenon does not appear in the estimated model for the other income groups and a Malthusian steady state does not yet exist in 1960 but comes into existence only later.

For *upper middle income* countries the results are summarized in column 3 of Table 5 and Figure A9. The production curve has decreasing returns to scale and the population curve has an inverted u-shape. A time trend shifts the population curve down and the production function upward. By implication, we have a shifting stable Malthusian steady state and an unstable Smithian one. For the year 2000 these countries are near the peak of the population

curve with $y = 4750$ and $N = .5$ billion. Malthusian behaviour therefore is essentially absent. The shifting time trend is stronger than the forces of stability and again dominates the process ensuring the growth of this group.

TABLE 5 OVER HERE

For *lower middle income* countries the results are summarized in the first column of Table 6 and in Figure A10 for the years 1960 and 2000 only. Both curves are falling in the relevant range as we have $y = 500$ and 1400 , and $N = 1.01$ and 2.5 billion in 1960 and 2000 respectively. Therefore Malthusian behaviour is absent. The time trend shifts both curves up. The time trend therefore works in favour of per capita income growth and population growth works against it. As the data show the positive impact of the time trend on income is larger than that of population growth and these countries are growing in per capita terms. Part of this though is due to growth in China, which is included in this group. There is no steady state, not even in the shifting form.

The *low-income* countries differ from the lower middle-income group in that the time trend has no impact on the production function. The results are shown in the last column of Table 6 and in Figure A11. The production function does not shift and therefore is plotted only once. Both curves have inverted u-shape and therefore Figure A11 is the one that resembles Foley's theoretical model of Figure 1 most closely. The population growth curve for 1960 allows for constant populations at incomes of about \$100 and \$650 and in 2000 for \$40 and \$740. The income levels for 1960 exist on the production function and therefore all four steady states of Foley's model as of Figure 1 exist. The maximum income of the production function is \$735.85. By implication the Smithian steady states vanish through the upward shift in the population growth curve around the year 2000. In the year 2060 the

Malthusian steady states have also vanished. Actually the prospects for the low-income countries are worse than Malthus thought. The population growth makes even a Malthusian steady state vanish. In the year 2001 the income in this sample is at \$480. This is in the falling part of the population growth curve. Therefore we again cannot find Malthusian behaviour. When population reaches a value of 6.5 billion, the income per capita reaches a maximum and thereafter declines. This value of the population is higher than the one we predicted for 2100 in the previous section and therefore will be reached only after 2100. This is a long time to go and therefore there is a big chance that population growth may induce a structural break of our estimates, which may bring about technical change or stop the shift in the population growth curve. This caveat is an optimistic one. The pessimistic ones are that either no structural break comes about and population remains growing or the growth comes from India, which is in this country set, but not from the other countries. Therefore we go to search for a different classification of countries in the next section. Before doing so we present some results for Sub-Saharan Africa.

For Sub-Saharan Africa the results are summarized in the last column of Table 6 and Figure A12. The figure resembles that of lower middle-income countries.⁴ There is no steady state and there is positive population growth with an upward shifting curve and there are decreasing returns in an upward shifting production function. Again the question is whether or not the time trend in the production function is able to outweigh the impact of population growth driving the economies into decreasing returns and decreasing per capita income. The data though are much less favourable than for lower middle-income countries. The Sub-Saharan region has had a positive growth rate for 1960-1974, then negative rates until 1994 and again positive ones in the most recent period. The average growth rate for the whole period is

⁴ Most countries are in the low-income group. However, there are four countries with an income above \$4000: Seychelles, South-Africa, Botswana, Mauritius and Gabon. They therefore belong to the group of upper-middle income countries (\$2936-9075). In the lower-middle income group (\$735-2935) are Cote d'Ivoire, Namibia, Swaziland.

an insignificant one of .1%. By implication, the problem of population growth outweighing the time trend is clearly present in Sub-Saharan Africa although there is no steady state and the population change behaves essentially in a Smithian way as income decreases population growth.

For the sets of poorer countries⁵ the question⁶ now is whether or not the positive growth results for lower middle income countries are based on the performance of China and for low income countries on those for India, and those for SSA are embellished by their richer countries (see footnote above).⁷ In the classifications used so far we cannot reject Foley's claim that history has been unfriendly to the ideas of Malthus in regard to Malthusian behaviour. But the problem of population pressure is still present for Sub-Saharan Africa but not for the larger groups including India and China. This can either be a consequence of the data or of the model. In this paper we keep sticking to the Foley's model and continue investigating the data.

TABLE 6 OVER HERE

5 The empirical income stagnation of low and some lower middle-income countries

For poor countries there is little doubt that population growth slows down GDP per capita growth whereas other factors enhance it. The crucial question in terms of finding the net effect therefore is to see how large the growth was and whether or not it was about zero for some countries. We plot the Kernel density estimates for GDP per capita first for a set of

⁵ The numbers of countries is 56 with high income, 34 with upper-middle income, 54 with lower-middle income, and 64 with lower income.

⁶ A quite different question relates to the differences in the properties of Figures A8-12 and their counterparts in Figures A13-17. The number of steady states and the shift properties are different. The reason probably is that the simplicity of the production side generates the low adjusted R-squared in Tables 4 and 5. The latter may signal an omitted variable bias. However, even if this is the case, the signs of the curves used in the interpretation may be unaffected by this as indicated by the significance of the coefficients.

⁷ Another reason of caution is that the number of countries reporting data is increasing. Taking growth rates may reflect growth in the delivery of data rather than growth of GDP per capita. Which way the bias goes may depend though on whether a newly reporting country is adding relatively more GDP or population.

countries for which we have data since 1960 in Figure A13 and in Figure A14 for all countries with data available in the respective years. The first approach follows a constant set of countries over time as suggested by Quah (1996) and the second looks at the updated distribution in each time period. The restricted data set confirms Quah's twin peak result. In the unrestricted data set the second peak is somewhat diluted. Both Figures show that there is always a peak at around 1000 constant 1995 dollars. It is hard to detect growth of the big peak with the mere eye. The local minimum following the big peak moves to a value of \$10000 during the decennia. By introspection we form the guess that there is little growth for countries below \$1200. Moreover, van den Berg (2001) suggests that per capita income at the end of the Malthusian phase for the world at 1820 of at \$651 according to Maddison (1995) or \$250 from Pritchett (1997). In Table 7, in the left part, we present counts telling how many countries had per capita incomes below these three values of \$250, \$651, and \$1200 in 1995 dollars⁸. As the number of countries for which data are available is increasing too, we take these numbers as percentage of all countries for which data are available. The percentage of countries, which is below the two Malthusian values of \$250 and \$651, has not improved during the period 1975-2000. For our value of \$1200 the percentage being above it has not improved since 1980. By and large the two descriptive methods of Figures A19 and A20 and Table 7 indicate that there may be a no-growth club of countries with incomes below \$1200. For the year 2002 there are 71 countries with per capita income below \$1200 (henceforth U1200). As India and China are among them we do not aggregate the data but rather consider a panel of countries in order to give less weight to the large countries.

TABLE 8 OVER HERE

⁸ However, in Pritchett's paper these are 1985 purchasing power equivalents, corresponding to \$70 in 1995 dollars. Even when transforming them into 1995 dollars there are only very few countries in that range. For our purpose of finding a set of countries with almost no growth we need a higher level. With about 9% of countries with data availability under \$250 in 1995 dollars we do not want to go to lower levels with lower percentages. Maddison's data were in 1990 dollars, which is a small difference and there is no need here to meet Maddison's number exactly.

In Table 8 we present the results from the regression of the (natural) logarithm of the GDP per capita of the U1200 countries on a country-specific constant and a common time trend. The coefficient for the time trend is well known to be the average growth rate. It is positive for the whole period 1960-2002 but as small as .0034, which is less than a half percent. It is a half percent during the last dozen of years 1991-2002. For all other periods it is negative. In particular, for the period 1970-2002 the result from Table 7 is confirmed saying that there is negative growth for a period of 33 years, including the two oil crises, the lost decade following the 1981/82 debt crisis, and also including the period from 1991-2002 with positive growth. This latter period does not compensate for the negative growth of the two previous decennia.

TABLE 9 OVER HERE

The results of a panel data estimate for the u1200 group are summarized in Table 9. It is a fixed effects estimate. The population change equation and the production function are estimated separately because the interdependence of the equation in the estimates for aggregated data in Tables 4 and 5 was very weak when comparing least squares and SUR (seemingly unrelated regression) estimates (both not shown). Moreover, we do not use a common factor restriction and we estimate in the SUR method because adding instruments in GMM leads to a ‘near singular matrix warning’ and the SUR method makes sure that the residuals of the countries are not taken as independent but rather the population growth (and the production function, each separately) equations are allowed to have correlated error terms. For the population growth equation this is plausible in particular because of the aids epidemic, and for the production function because of the interrelated business cycles and growth processes. Moreover, in the estimation of the production function we can improve the

estimate considerably by allowing for country-specific time trends. When having the estimation results we extract the coefficient of population and GDP per capita respectively as a common factor from the level variables, set the error-correction term equal to zero and get the following equation in terms of the coefficients of Table 9:

$$dN = \frac{798000}{0.66} + \frac{-842.87}{0.66} y + \frac{0.36}{0.66} y^2 + \frac{3699}{0.66} \times t$$

$$y = \frac{47.26}{0.081454} + \frac{-6.88 \times 10^{-07}}{0.081454} N + \frac{3.28 \times 10^{-16}}{0.081454} N^2 + \frac{.68}{0.081454} t$$

In the second equations and in the plot in Figure A16 we use the average value of the time trends.⁹ In Table 9 we have given the intervals in which they are in the estimates. Figure A16 shows that population behaviour in the u1200 group is essentially Smithian and there are decreasing returns to scale. Therefore Malthus' idea does not come out here but the population problem is of course keeping growth down. The panel average of per capita income is $y = \$475$ and $N = 38\text{mln}$, and for the years 2000-2002 they are $y = \$494$ and $N=52.5\text{mln}$. The latter average number indicates that the 71 countries account for 3.5 billion people, which is more than half of the world population in the no growth club. However, once India and China are taken out, the club size is 1 billion people. On the other hand then the income limit of \$1200 might be larger as well. Again, an income change from \$475 to \$494 within twenty years (from the middle of the panel in 1981 to 2001 for the restricted panel with years 2000-2001, this is one dollar a year and $1/5^{\text{th}}$ of a percent) confirms that there is little growth in GDP per capita but a high one in population growth. The prospects are of course

⁹ The result is decreasing returns with an upward shift of the production function. When using a common time trend results are much worse statistically and give increasing returns with a downward shift. This latter results is hard to interpret and the statistical results suggest that we should not use a common time trend.

worst for countries with negative time trends. The distribution of the coefficients of the time trend variable is plotted in Figure A16. The countries with the negative time trends are: Bolivia, Burundi, Central African Republic, Comoros, Congo DR, Cote d'Ivoire, Eritrea, Gambia, Georgia, Guinea-Bissau, Haiti, Kiribati, Kyrgyz Rep., Liberia, Madagascar, Mauritania, Moldova, Mongolia, Nicaragua, Niger, Papua New Guinea, Sierra Leone, Solomon Island, Tajikistan, Togo, Vanuatu, West Bank and Gaza, Zambia, Zimbabwe. This is roughly what you might expect and again confirms that Foley's model is not only simple but also very realistic.

6. Summary and conclusion

In section three, we have shown that Foley's model produces reasonable forecasts for the population growth of the aggregated country groups according to the World Bank income classification. In section four, we found plausible estimates of the two-equation system version of Foley's model. Malthusian behaviour is hard to detect unless one accepts that it is present in high-income countries whereas it is clearly absent in all other groups of countries. An interpretation of children having a high-income elasticity, or, richer parents can afford to have more children, seems more plausible. Foley's increasing returns result is found on the world level and for low-income countries. For the other group we find decreasing returns to population growth. As aggregate data include India in the low income group and China in the lower-middle income group one gets even the impression here that a problem of population growth is present only in Sub-Saharan Africa. Therefore we continue with even more disaggregated data. In section three we show that (i) the density distribution of world income has a stagnant peak around \$1000 and (ii) that the percentage of countries that is below the income levels of \$250 and \$650 has not improved since 1975 and (iii) for the level of \$1200 there is stagnation since 1980. The suspicion that the u1200 group is a no-growth club – with

the exceptions of individual countries of course – is confirmed by panel regression analysis. For the period 1975-2002 the panel average growth rate is negative in spite of the fact that it is positive for the years 1991-2002. The estimation of Foley's model for the u1200 group shows a curvature that resembles that of the lower middle-income countries and Sub-Saharan Africa. However, the curves show lower population change and lower incomes because the large and rich countries get a higher weight in a panel compared to aggregated data. In the u1200 group Malthusian behaviour is absent, but population growth outweighs the growth effects of a positive time trend under conditions of decreasing returns to scale. This seems to be closer to Ricardo's version of the population growth problem than to that of Malthus.

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Table 1 Least squares estimation and test of Foley's Model

Dependent variable: Population change

Variable	Group	World	HI	UMI	LMI	LI
constant		-4.26E+09 (-9.757)	4.94E+09 (-1.772)	1.29E+08 (-7.412)	-3.19E+09 (-10.592)	-5.93E+08 (-4.615)
Pop(-2)		3.934 (-9.839)	-25.69 (-1.814)	-1.507 (-7.361)	8.312 (-10.63)	1.647 (-4.656)
Pop(-2) ²		-1.28E-09 (-9.769)	4.98E-08 (-1.858)	6.68E-09 (-7.540)	-7.86E-09 (-10.571)	-1.45E-09 (-4.552)
Pop(-2) ³		1.78E-19 (-9.682)	-4.23E-17 (-1.893)	-1.27E-17 (-7.580)	3.25E-18 (-10.508)	4.89E-19 (-4.498)
Pop(-2) ⁴		-9.77E-30 (-9.695)	1.32E-26 (-1.911)	8.73E-27 (-7.507)	-5.15E-28 (-10.56)	-6.68E-29 (-4.506)
Time		-	-	-		
time ²		565861.7 (-7.897)	33894.73 (-3.898)	-	172386.6 (-7.583)	300334 (-4.500)
adjR ²		0.917	0.729	0.938	0.839	0.991
JarqBera F-Prob		0.305	0.225	0.698	0	0.012
Breusch-Godf F-Prob		0.057 (1)	0.087 (1)	0.040 (1)	0.840	0.003
White hetsc. F-Prob.		0.033	0.796	0.496	0	0.030
Ram. Res. T. F-Prob		0.0026(2)	0.13 (2)	0.0069	0.129(1)	0.060(1)
Cusum		5%	5%	5%	5%	5%
Cusum squared		>5% (1990)	>5%	>5% (96-99)		5%>>5%
No. Osf		4,<10%	5,<15%	4, <15%	4,<15%	11, <15%
No. nsf		6, <15%	10<10%	15, <15%	4,<15%	30, <15%

Legend:

t-values according to the Newey-West HAC in parantheses

Cusum (sq) cumulated sum of (squared) residuals

No. osf Number of observations of one step forecast test doubting parameter constancy

No. nsf Number of observations of n step forecast test doubting parameter constancy

Table 2 Least squares estimation for forecasting with Foley's Model

Dependent variable: Population change

Variable	Group	World	HI	UMI	LMI	LI (1)	LI (2)
constant		-4.46E+08	-1.02E+08	1.99E+08	-5.37E+08	7.63E+08	3.95E+08
		-(1.218)	-(1.583)	-(5.986)	-(1.401)	(5.547)	(2.642)
Pop(-2)		0.926	0.355	-1.347	1.661	-1.013	-0.566
		-(2.980)	-(1.879)	-(5.907)	-(1.625)	-(5.397)	-(2.561)
Pop(-2) ²		-4.31E-10	-2.84E-10	2.68E-09	-1.75E-09	2.90E-10	2.12E-10
		-(4.189)	-(2.037)	-(6.072)	-(1.769)	(5.154)	(2.758)
Pop(-2) ³		6.99E-20	-	-2.44E-18	7.65E-19	-5.37E-20	-3.58E-20
		-(4.534)	-	-(6.282)	-(1.852)	-(5.578)	-(2.971)
Pop(-2) ⁴		-4.11E-30	-	-	-1.24E-28	-	-
		-(4.806)	-	-	-(1.939)	-	-
Time		16900021	-	2668011	2861633	13645096	6755729
		-(6.200)	-	-(5.875)	-(2.671)	(5.667)	(2.661)
time sq		-	14561.14	-	-	125761.1	
		-	-(2.640)	-	-	(3.923)	
d(po(-1))		-	-	-	0.406567	-	
					-(3.771)	-	
adjR ²		0.889	0.689	0.889	0.879	0.992	0.986
JarqBera F-Prob		0.302	0.478	0.785	0.161	0.00058	0
Breusch-Godf F-Prob		0.000044	0.011 (1)	0.000394	0.098 (1)	0.012 (1)	0.0018 (5)
White hetstc. F-Prob.		0.167	0.110	0.000001	0.000001	0.087	0.0016
Ram. Res. T. F-Prob (Nft)	0 (2)	0.068 (1)	7.4E-05 (1)	0.01453 (2)	0.109457 (4)	0.000252(2)	
Cusum	5% (1993)	5%	5%	5%	5%	5%	5%
Cusum squared	>5%	5%	>5% (81-99)	>5%	>5%(75-92)	>5% (1975-92)	
No. Osf	9, <15%	3, <10%	4, <5%	11, <15%	11, <10%	11, <10%	
No. nsf	23, bef.1995	2, 5%	29, <15%	17, bef.1988	25, <1%	all,10%	
Pop 2100		8594022559	1653189186	779214678	2.76E+09	5.31E+09	4.29E+09
Sum Pop 2100		1.0502E+10	with Li(1), or	9.48E+09	with Li(2)		

Legend:

t-values according to the Newey-West HAC in parantheses

Cusum (sq) cumulated sum of (squared) residuals

No. osf Number of observations of one step forecast test doubting parameter constancy

No. nsf Number of observations of n step forecast test doubting parameter constancy

Table 3 Group year	Population Predictions using Foley's Model					
	World	High Income	Upp.mid.inc.	Low.mid.inc.	Low.inc.(1)	Low.inc.(2)
2000 (actual)	6,056,112,696.23	949,896,174.00	497,195,284.22	2,145,637,180.95	2,462,084,098.57	2,462,658,633.35
2010	6,673,136,074.39	1,017,458,213.70	547,760,505.98	2,283,931,072.23	2,885,148,912.33	2,853,091,299.82
2020	7,084,478,300.97	1,086,696,920.90	588,763,058.98	2,378,591,159.29	3,264,439,342.15	3,144,078,351.12
2030	7,392,197,245.42	1,156,702,687.78	623,163,295.48	2,452,194,812.69	3,601,284,377.31	3,367,053,493.23
2040	7,640,497,583.52	1,227,130,933.62	652,873,992.33	2,513,062,564.21	3,903,947,536.68	3,550,668,271.34
2050	7,849,988,853.92	1,297,824,694.63	679,110,368.67	2,565,292,186.96	4,180,004,454.13	3,708,408,237.38
2060	8,032,017,702.13	1,368,698,225.62	702,674,659.67	2,611,234,056.18	4,435,189,624.50	3,847,598,458.73
2070	8,193,520,231.41	1,439,699,791.34	724,119,674.96	2,652,365,524.83	4,673,695,699.57	3,972,739,595.64
2080	8,339,057,384.02	1,510,796,056.25	743,841,208.93	2,689,726,994.76	4,898,604,706.66	4,086,818,622.13
2090	8,471,794,687.36	1,581,964,469.71	762,132,017.76	2,724,009,162.66	5,112,188,830.36	4,191,929,665.71
2100	8,594,022,557.63	1,653,189,185.70	779,214,677.89	2,755,593,817.52	5,313,105,079.96	4,289,603,023.47
RMSE (a)	4,841,278.00	1,419,178.00	344,217.00	3,782,967.00	1,172,513.00	1,443,878.00
MAE (b)	3,939,963.00	1,144,408.00	279,121.00	3,194,104.00	815,150.00	1,154,588.00
MAPE (c)	0.08742	0.13845	0.07557	0.19886	0.04544	0.06868
Theil In.Co (d)	0.00052	0.00085	0.00045	0.00112	0.00033	0.00041
Bias Prop.(e)	0.00027	0.00151	0.00299	0.00054	0.00003	0.00023
Var.Prop.(f)	0.00050	0.00160	0.00674	0.00147	0.00012	0.00063
Cov.Porp.(g)	0.99923	0.99689	0.99027	0.99798	0.99986	0.99915

(a) RMSE is root mean squared error: the root of average of the squared deviations of the predicted from the actual value during the forecast part of the sample period.

(b) MAE is mean absolute error: the average of the absolute values of the deviations of the predicted from the actual value during the forecast part of the sample period.

(c) MAPE is mean absolute percentage error: the deviations in MAE are divided by the actual value.

(d) The Theil inequality index divides the RMSE by the sum of two roots: the root of the mean of the squared forecasted values and the root of the mean squared actual values.

(e), (f),(g): percentage of RMSE telling how far the mean and variation of the forecast is from the mean (variation) of the actual value, and an unexplained rest respectively.

Table 4

Recent forecasts		period	Forecasts from the 1980s for 2100	
UN	5.5-14	2100	UN 1983	7-14
IIASA	9	2070	UN 1988	10.8
Foley	7.5-8.5	2100	World Bank 1988	10.5

This paper:

**For aggregate world data 8.6;
by income groups 10.5 or 9.5**

Sources: Foley (2000) using Loess fits; Documents (2003); Lee (1991); this paper

Table 5 Results of System Regression for High and Middle Income Countries

Dependent variable: (First difference of) population change

Coefficient

(t-value)	Variable\Group	World ^(a)	high inc. ^(b)	upp.mid.inc. ^(c)
c(1)	constant	-8848829.00 (-.33)	6335751.00 (6.18)	-10132.10 (-.01)
c(2)	GDPpc(-2)	38782.64 (3.87)	237.60 (2.06)	3803.16 (7.32)
c(3)	(GDPpc(-2)) ²	-4.22 (-4.44)	0.02 (11.62)	-0.43 (-5.53)
c(4)	time	-	-508827.70 (-9.90)	-49755.87 (-4.52)
c(9)	error corr.term	-0.10 (-2.53)	-0.81 (-17.58)	-0.18 (-2.02)

Dependent variable: (Change of) GDP pc

c(5)	constant	1048.29 (1.68)	98026.44 (2.73)	12555.69 (4.27)
c(6)	population	0.00 (2.78)	0.00 (-2.13)	0.00 (-3.24)
c(7)	Population ²	0.00 (.21)	0.00 (1.31)	0.00 (-2.26)
c(8)	time	-	910.15 (8.45)	768.19 (3.95)
c(10)	error corr.term	-0.25 (-6.31)	-0.16 (-4.74)	-0.15 (-2.51)
Adj. R ²		0.11/.16	0.335/.075	0.07/.25
DW		1.40/1.50	1.93/1.39	2.08/1.58
J-statistic		0.254	0.281	0.205
no.obs. = n		37/36	36/38	38/40
nJ (1st eq.)<chi-sq.(d.f.),5%lev. ^(d)		37x.25<16(9)	36x.28<21(12)	38x.2<11(5)
nJ (2nd eq.)<chi-sq.(d.f.),5%lev.		36x.25<18(10)	38x.28<21(12)	40x.2<11(5)

(a) Instruments: 1st eq.: c d(N(-1)) y(-3) y(-3)² d(y(-2)) d(y(-2))² d(N(-2)) d(y(-3)) d(y(-3))² d(N(-3)) d(y(-4)) d(y(-4))² d(N(-4))2nd eq.: c y(-1) N(-1) N(-1)² d(N) d(N)² d(y(-3)) d(N(-2)) d(N(-2))² d(y(-4)) d(N(-3)) d(N(-3))² d(y(-5)) d(N(-4))(b) Instruments: 1st eq.: c d(N(-1)) y(-3) y(-3)² t d(y(-2)) d(y(-2))² d(N(-2)) d(N(-3)) d(N(-4)) d(y(-3)) d(y(-3))² d(y(-4)) d(y(-4))² d(y(-5)) d(y(-5))²2nd eq.: c y(-1) N(-1) N(-1)² t d(N) d(N)² y(-2) y(-3) y(-4) d(N(-1)) d((N(-1))²) d(N(-2)) d((N(-2))²) d(N(-3)) d((N(-3))²)(c) Instruments: 1st eq.: c d(N(-1)) y(-3) y(-3)² t d(y(-2)) d(y(-2))² d(N(-2)) d(y(-3)) d(y(-3))²;2nd eq.: c y(-1) N(-1) N(-1)² t d(N) d(N)² y(-2) d(N(-1)) d((N(-1))²)

(d) The number for the degrees of freedom is equal to the number of constraints in the equation plus the number of instruments going beyond the number of regressors.

Table 6

Results of System Regression for lower middle and low income countries and Sub-Saharan Africa

Dependent variable: (First difference of) population change

Coefficient

(t-value)	Variable\Group	low.mid.inc ^(d)	lower inc. ^(e)	SSA ^(f)
c(1)	constant	153000000.00 (1.85)	-23788013.00 (-3.48)	9475210.0 (1.98)
c(2)	GDPpc(-2)	-274399.90 (-5.91)	281118.30 (7.62)	-18839.6 (-1.21)
c(3)	(GDPpc(-2)) ²	140.18 (6.17)	-380.30 (-11.22)	18.9 (1.50)
c(4)	time	4815680.00 (1.57)	452914.40 (4.97)	267950.2 (48.37)
c(9)	error corr.term	0.04 (1.68)	-0.35 (-8.04)	-0.5 (-19.05)

Dependent variable: (Change of) GDP pc

c(5)	constant	1898.40 (2.29)	30.54 (1.82)	2623.2 (46.71)
c(6)	population	0.00 (-1.69)	0.00 (11.94)	0.0 (-34.89)
c(7)	population ²	0.00 (-.48)	0.00 (-3.42)	0.0 (33.48)
c(8)	time	68.01 (4.60)	- (33.65)	80.7 (33.65)
c(10)	error corr.term	-0.18 (-3.00)	-0.41 (-7.49)	-0.5 (-23.04)
Adj. R ²		.08/.096	.095/.14	.12/.31
DW		1.26/1.09	1.4/1.35	1.67/1.40
J-statistic		0.247	0.224	0.278
no.obs. = n		33/35	37/38	35/37
nJ (1st eq.)<chi-sq.(d.f.),5%lev ^(g)		33x.25<12.5(6)	37x.224<16.9(9)	35x.28<28(18)
nJ (2nd eq.)<chi-sq.(d.f.),5%lev.		35x.25<12.5(6)	38x.224<19.6(11)	37x.28<28(18)

(d) Instruments: 1st eq.: c d(N(-1)) y(-3) y(-3)² t d(y(-2)) d(y(-2)²) d(N(-2)) d(y(-3)) d(y(-3)²);2nd eq.: c y(-1) N(-1) N(-1)² t d(N) d(N²) y(-2) d(N(-1)) d((N(-1))²)(e) Instruments: 1st eq.: c d(N(-1)) y(-3) y(-3)² t d(y(-2)) d(y(-2)²) d(N(-2)) y(-4) y(-4)² d(N(-3)) y(-5) y(-5)²;2nd eq.: c y(-1) N(-1) N(-1)² d(N) d(N²) y(-2) N(-2) N(-2)² y(-3) N(-3) N(-3)² y(-4) N(-4) N(-4)²(f) Instruments: 1st eq.: c d(ssa(-1)) yssa(-3) yssa(-3)² t d(yssa(-2)) d(yssa(-2)²) d(ssa(-2)) d(yssa(-3)) d(yssa(-3)²) d(ssa(-3)) d(yssa(-4)) d(yssa(-4)²) d(ssa(-4)) d(yssa(-5)) d(yssa(-5)²) d(ssa(-5)) d(yssa(-6)) d(yssa(-6)²) d(ssa(-6)) d(yssa(-7)) d(yssa(-7)²)2nd eq.: c yssa(-1) ssa(-1) ssa(-1)² t d(ssa) d(ssa²) yssa(-2) d(ssa(-1)) d(ssa(-1)²) yssa(-3) d(ssa(-2)) d(ssa(-2)²) yssa(-4) d(ssa(-3)) d(ssa(-3)²) yssa(-5) d(ssa(-4)) d(ssa(-4)²) yssa(-6) d(ssa(-5)) d(ssa(-5)²)

(g) The number for the degrees of freedom is equal to the number of constraints in the equation plus the number of instruments going beyond the number of regressors.

Table 7

Number of countries with GDP per capita (constant 1995\$) not higher than

year	\$250	\$651	\$1200	obs.available
1960	14	39	55	101
1970	12	40	59	118
1973	11	37	54	120
1975	11	35	54	123
1980	12	37	55	140
1990	15	50	69	174
2000	16	52	70	178

Percentage of countries with GDP per capita ^(a)

year	< \$250	< \$651	< \$1200
1960	0.138	0.386	0.544
1970	0.101	0.338	0.5
1973	0.091	0.308	0.45
1975	0.089	0.284	0.439
1980	0.085	0.264	0.392
1990	0.086	0.287	0.396
2000	0.089	0.292	0.393

(a) Percentage of observations available.

Source: author's counts based on WDI 2003

Table 8 **Average growth rate of countries with GDP per capita below \$1200^(c)**

Period	1960-2002	1965-2002	1970-2002	1980-2002	1991-2002
growth rate ^(a)	0.0034	0.00156	-0.0007	-0.0024	0.005
(t-value) ^(b)	4.98	2.2	-1.09	-3.25	1.96
adj.R-square	0.77	0.786	0.8	0.82	0.94
Countries	71	71	71	71	71
periods	43	38	33	28	12
total obs.	2342	2147	1937	1712	845

(a) The estimate is done with the fixed effect mode using country specific intercepts but a common coefficient for the time trend.

(b) t-values are based on panel corrected standard errors (PCSE) for cross-section SUR (see Beck and Katz 1995).

(c) 1995 dollars

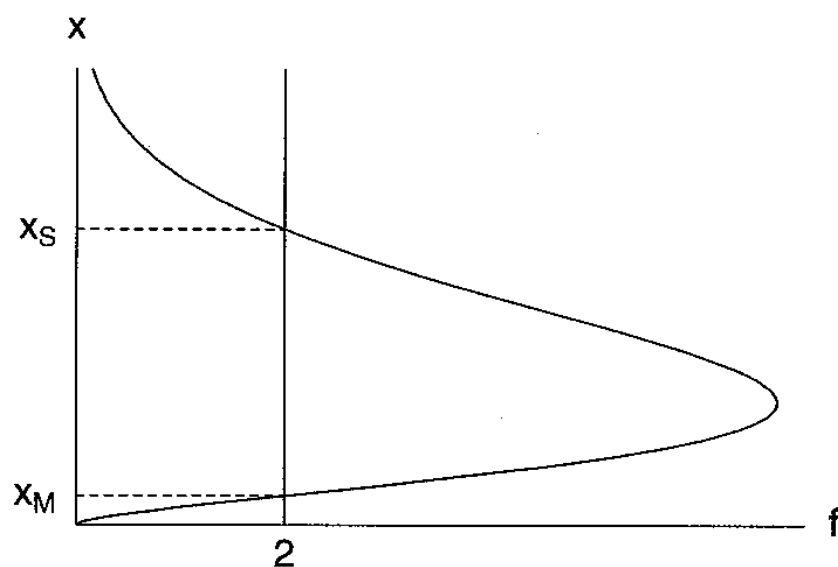
Table 9**Results of Panel Regressions for Countries with GDP per capita below \$1200**

Depend. variable: Change of Population change Change of GDP per capita

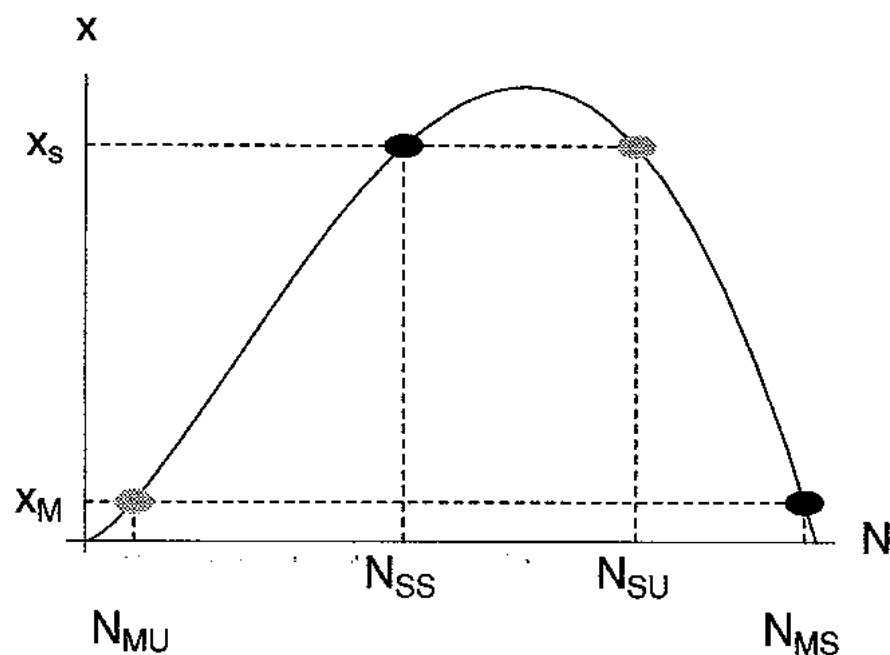
Variable	Coefficient (t-value) ^a	Coefficient (t-value)
Constant ^b	797884.00 (4.74)	47.26 (2.55)
d(pop(-1))	-0.66 (-5.69)	-0.08 (-2.87)
GDPpc(-3)	-842.87 (-2.23)	-0.00 (-3.62)
(GDPpc(-3))^2	0.36 (2.32)	0.00 (4.46)
time	3699.17 (2.33)	(-31.8, .68, 23.5) ^c
d(GDPpc(-2))	-1114.73 (-1.75)	0.00 (2.29)
d((GDPpc(-2))^2)	0.43 (1.81)	0.00 (-2.27)
Adjusted R-squared	0.32	0.13
DW	2.32	1.34
no.obs. = n	2129 (40x71)	2271 (42x71)

- (a) t-values in parantheses (from panel corrected standard errors with cross-section SUR)
(b) The fixed effects of both regressions are not shown.
(c) For the country specific time trends in the second regression we present the minimum, average, and maximum value.

Figure 1



The theoretical income–fertility relation has both an upward sloping segment, corresponding to Malthus’ assumption of rising fertility with income, and a downward sloping segment, corresponding to the demographic transition in which fertility falls with income. There are two equilibrium levels of per-capita output, at which total fertility equals 2, the Malthusian equilibrium x_M , and the Smithian equilibrium, x_S .

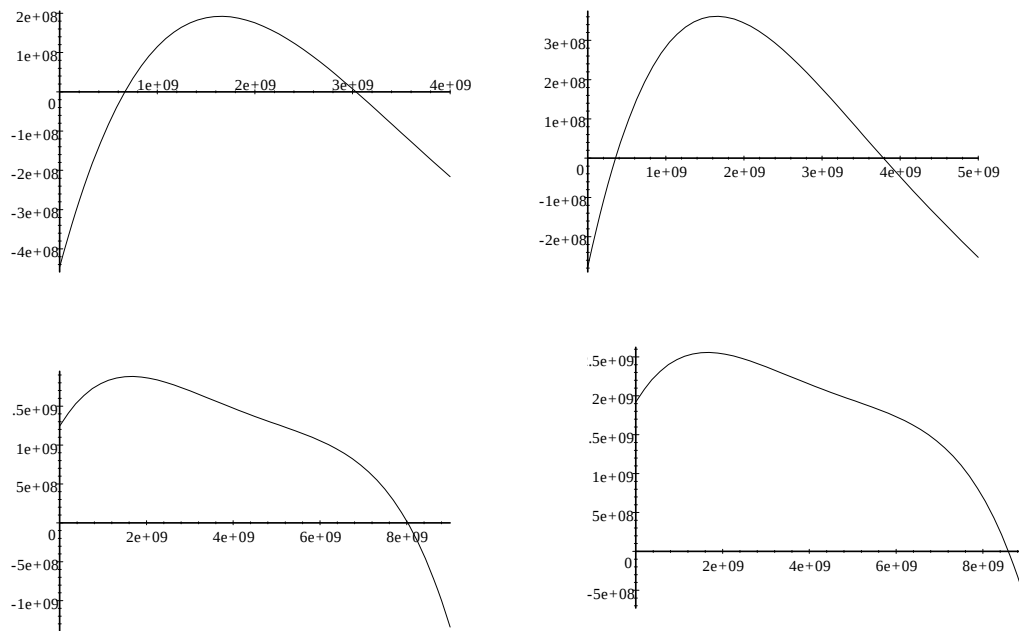


When the population–per-capita output relation has both a rising portion, representing the effects of the division of labor, and a falling portion, representing the effects of diminishing returns, there are potentially two Malthusian and two Smithian equilibria. The low-population Malthusian equilibrium and the high-population Smithian equilibrium are unstable, while the low-population Smithian equilibrium and the high population Malthusian equilibrium are stable.

Source: Foley 2000

Appendix: Figures

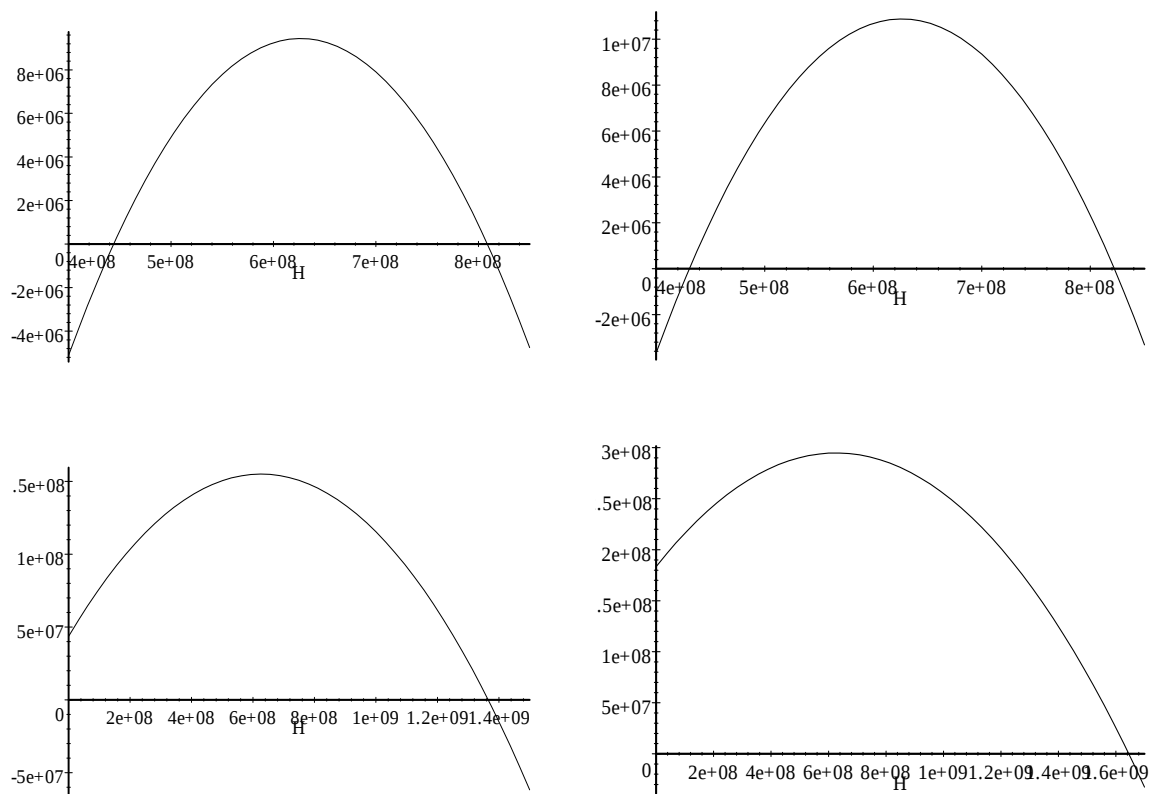
Figure A2



Population for the world shifts upward according to Foley's model. The population levels are (predicted to be) 3.06, 3.68, 8.03 and 8.59 billion people in 1960, 1970, 2060 and 2100.

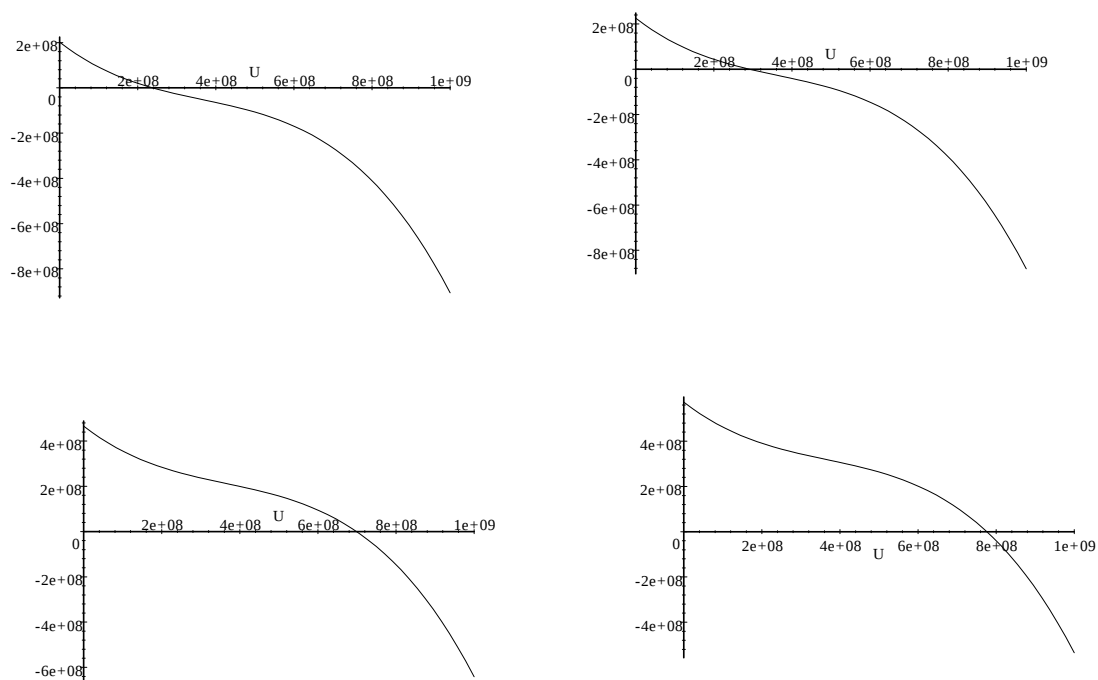
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Figure A3



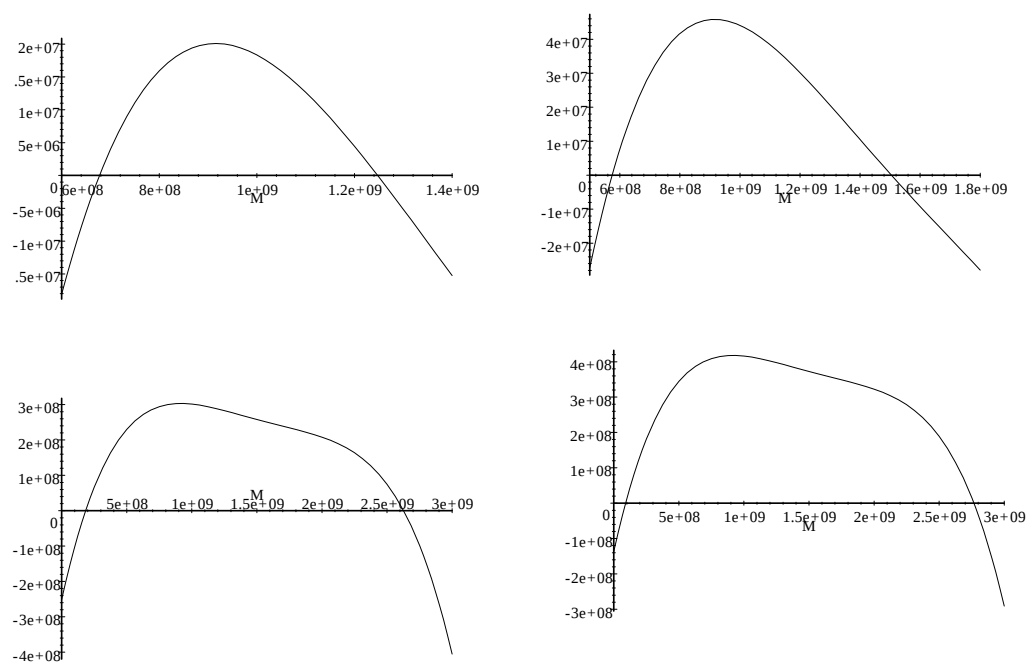
The population growth curve for the high-income countries has moved up and has become broader. The population in 1960, 1970, 2060 and 2100 is (predicted to be) .6772, .758, 1.37, 1.65 billion.

Figure A4



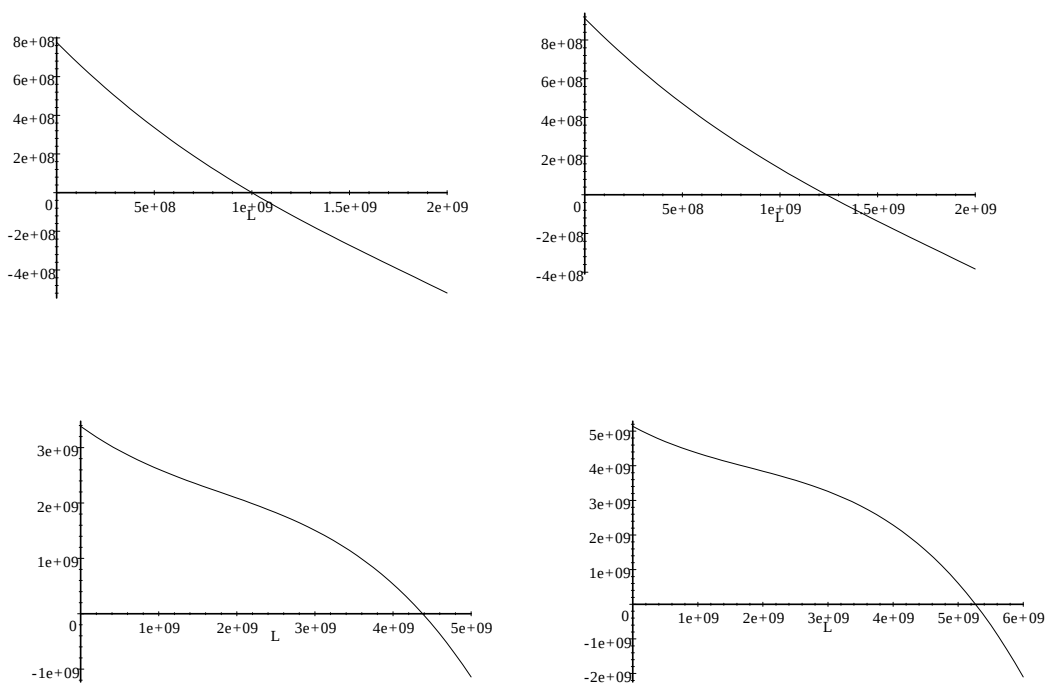
The population dynamics of upper middle-income countries has only one moving steady state shifting to the right. The population in 1960, 1970, 2060 and 2100 is (predicted to be) .236, .294, .784, and .931 billion.

Figure A5



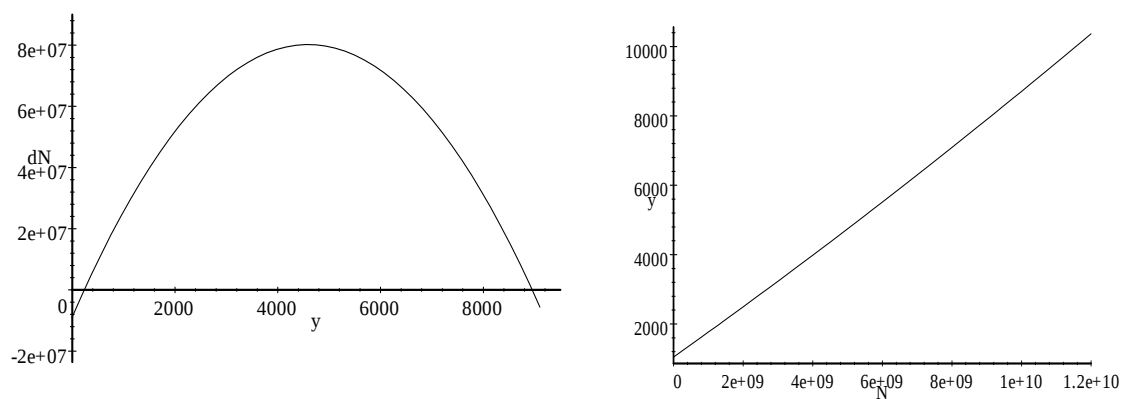
The population dynamics of the lower-middle-income countries has a stable and an unstable steady state. The whole curve shifts up over time. The population in 1960, 1970, 2060 and 2100 is (predicted to be) 1.1, 1.35, 2.65, 2.76 billion.

Figure A6



For low-income countries there is only one moving steady state shifting to the right. The population in 1960, 1870, 2060 and 2100 is (predicted to be) 1.01, 1.27, 4.4, 5.31 billion.

Figure A7: Estimation for aggregate world data



For aggregate world data we find increasing returns to scale, an unstable Malthusian and a stable Smithian steady state and no time trends. Per capita income for 2000 is $y = 4750$.

Figure A8: High income countries in 1960, 2000, 2060, 2100

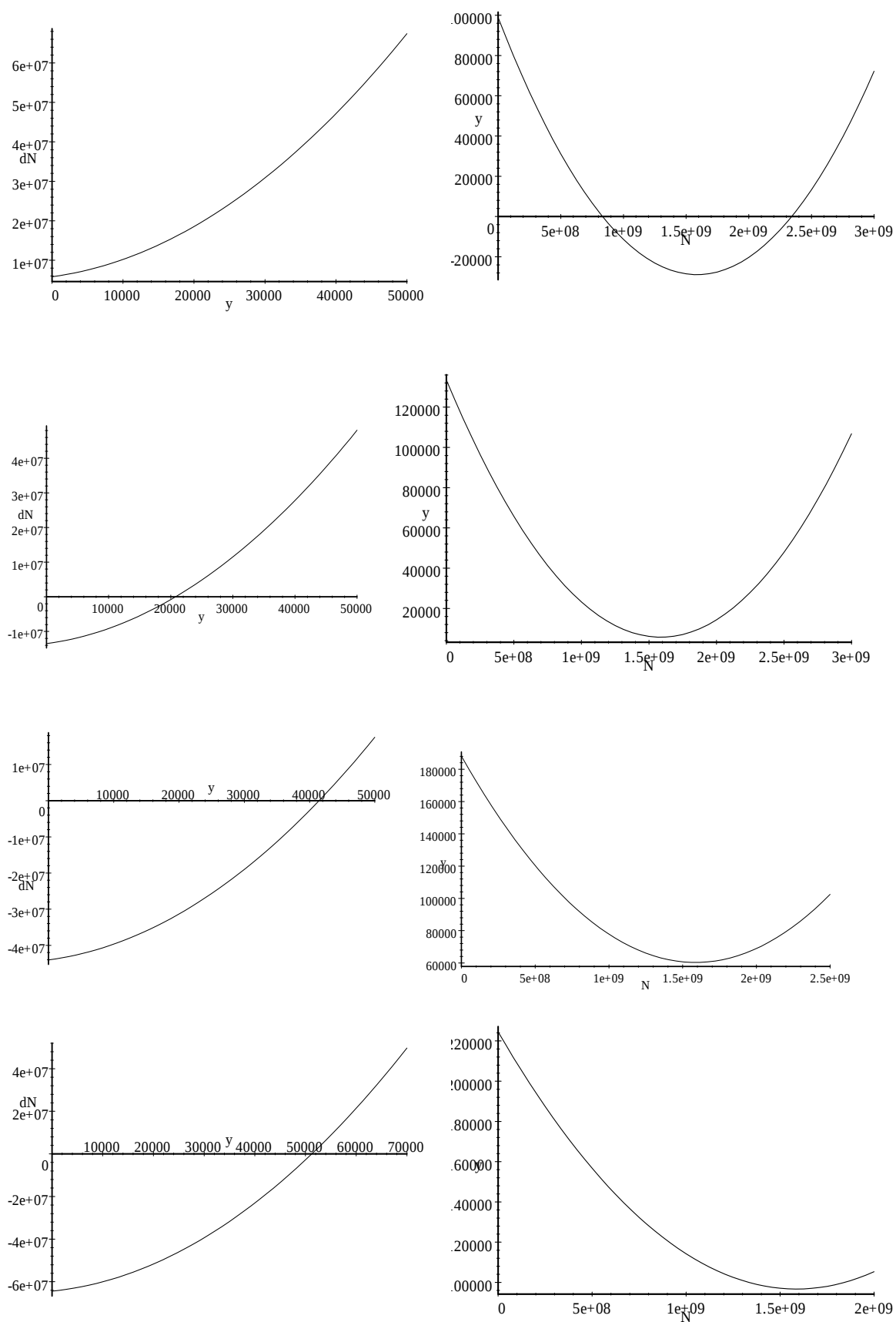


Figure A9: Upper middle income countries in 1960, 2000, 2060 and 2100

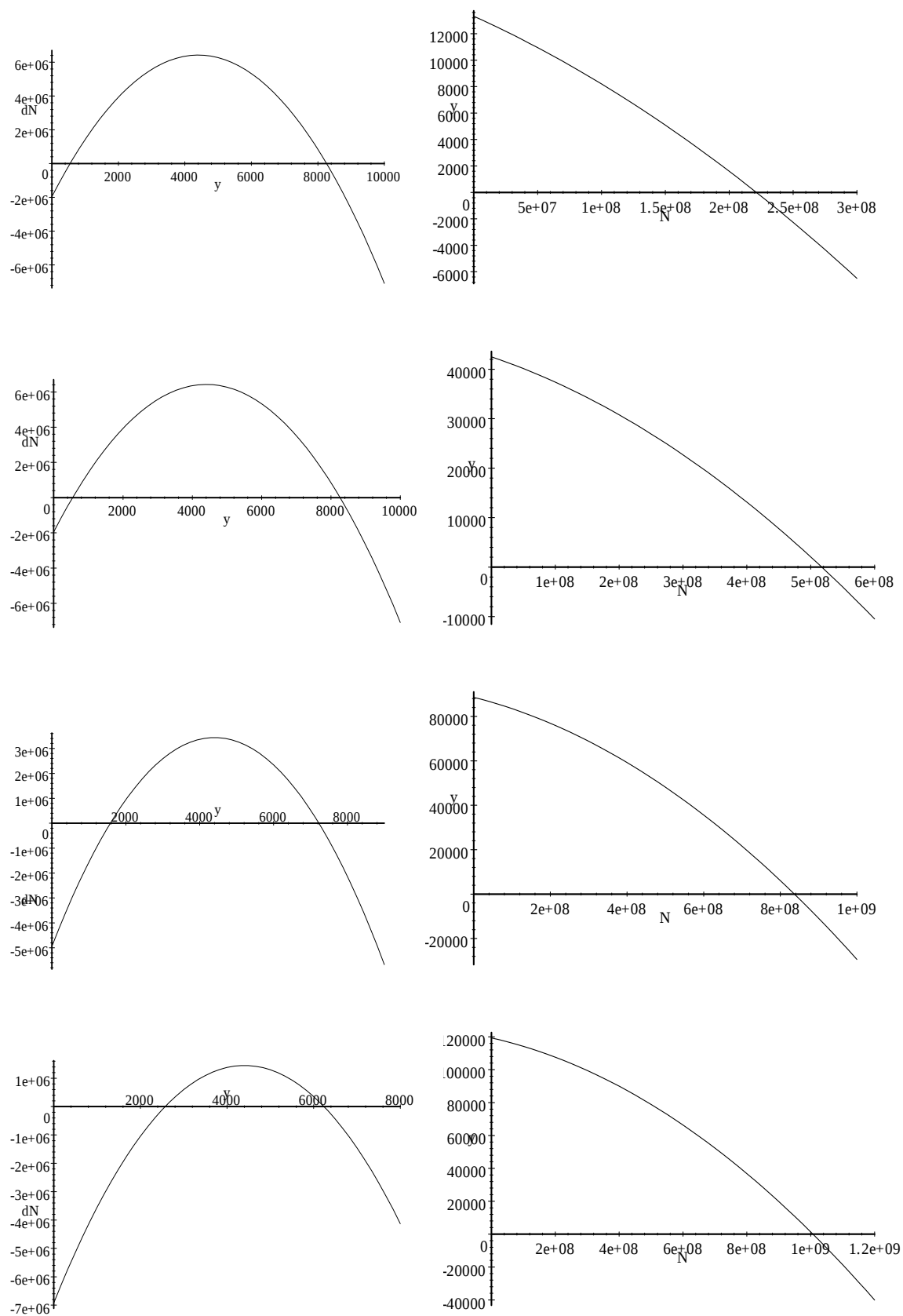


Figure A10: Lower middle income countries 1960, 2000

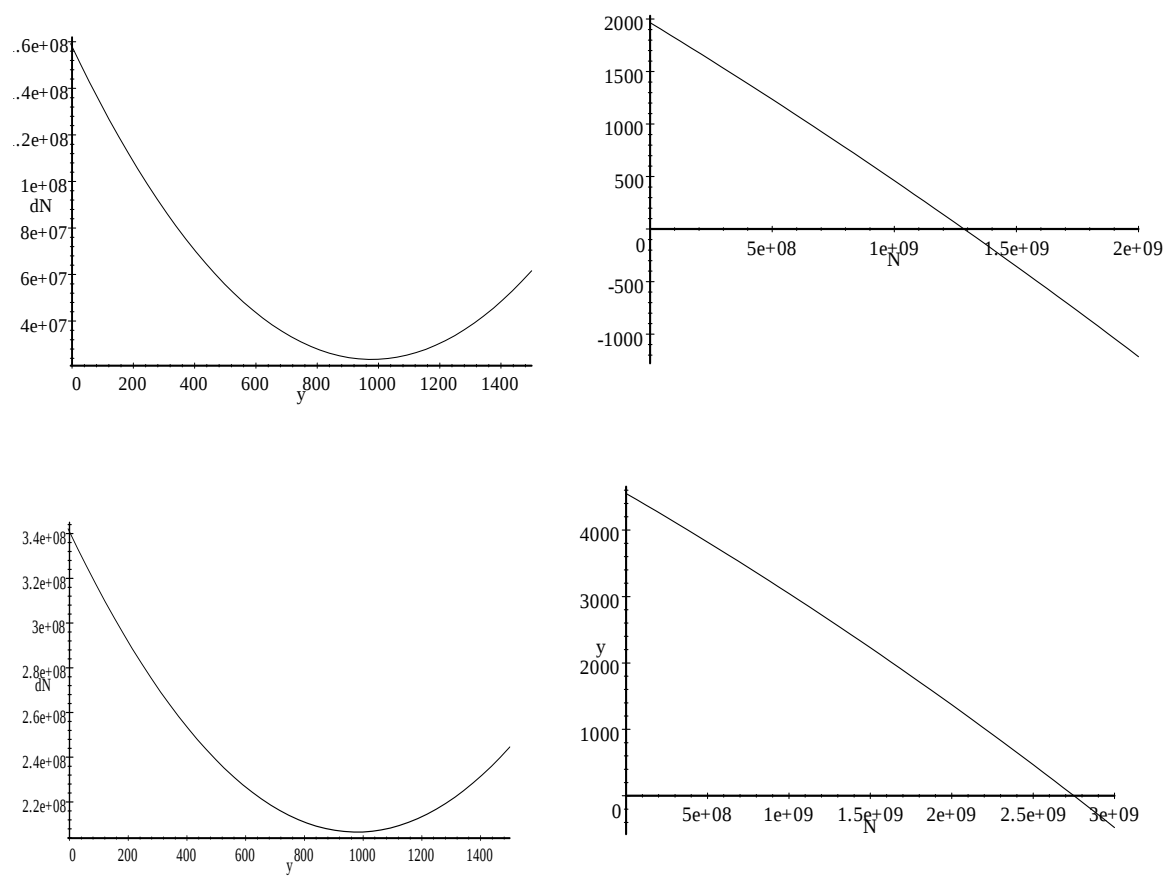


Figure A11: Low income countries in 1960, 2000, 2060, 2100.

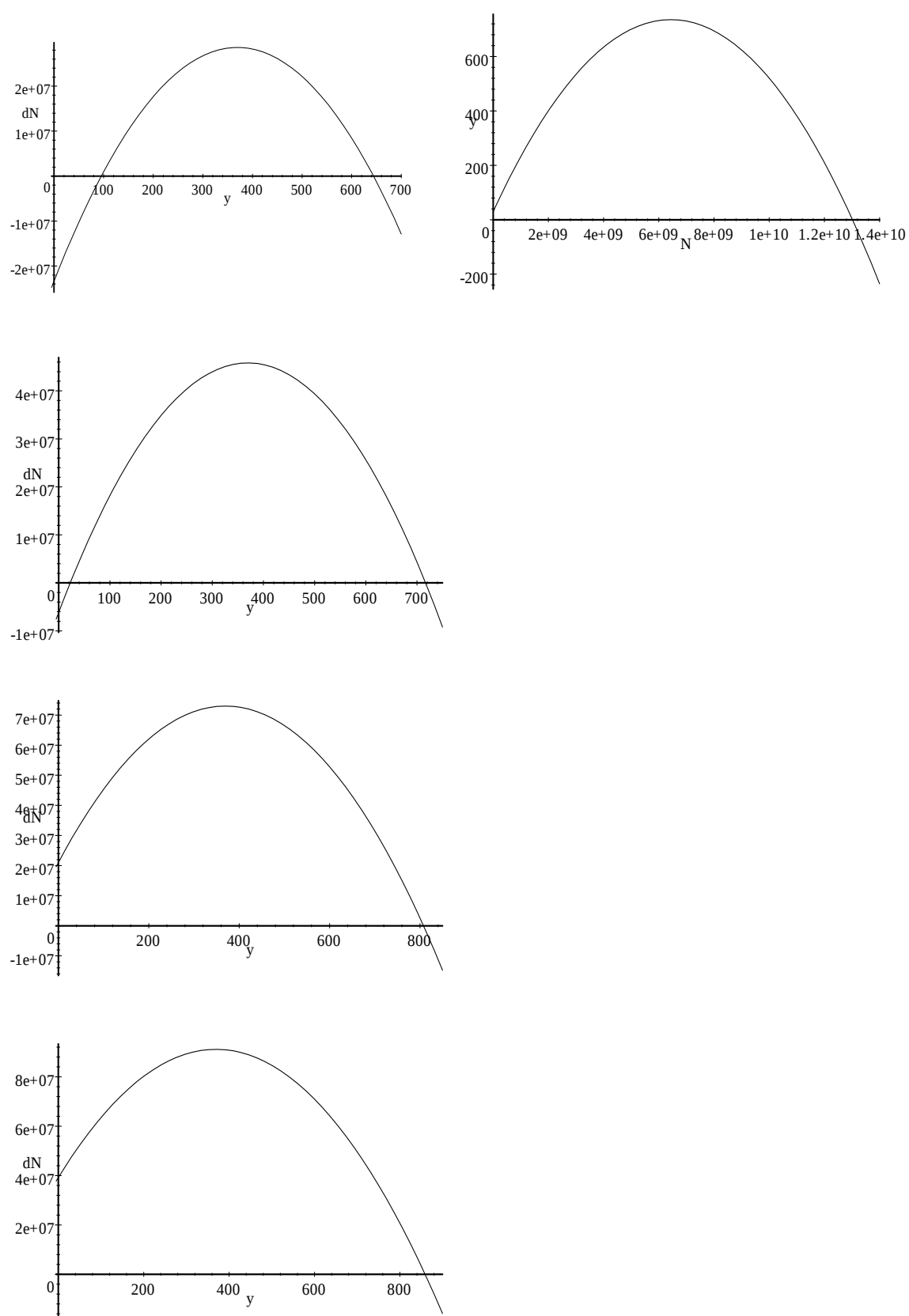


Figure A12: Sub-Saharan Africa 1960, 2000, 2060

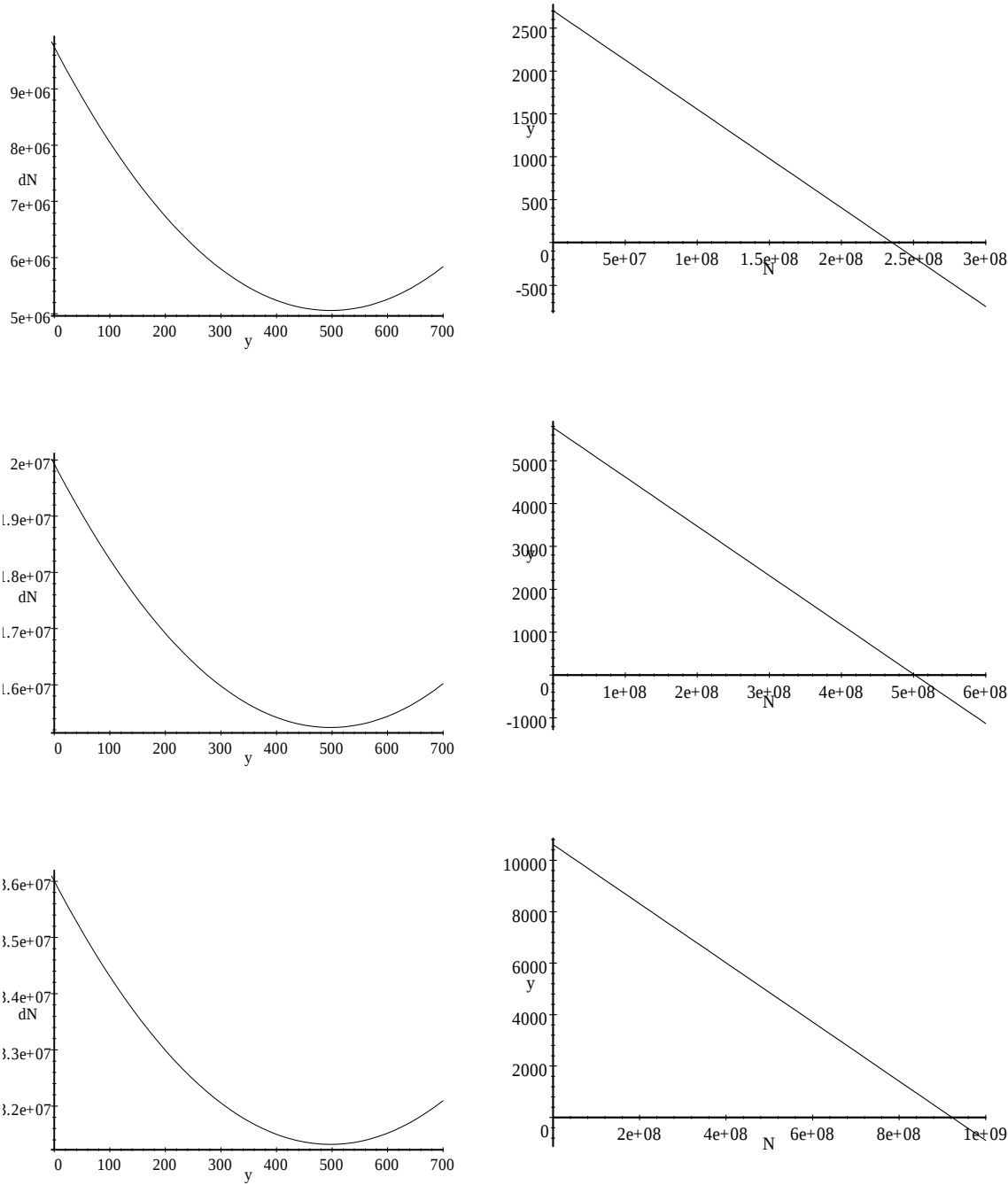


Figure A13: GDP Per Capita (constant 1995 US dollars) – *restricted dataset*

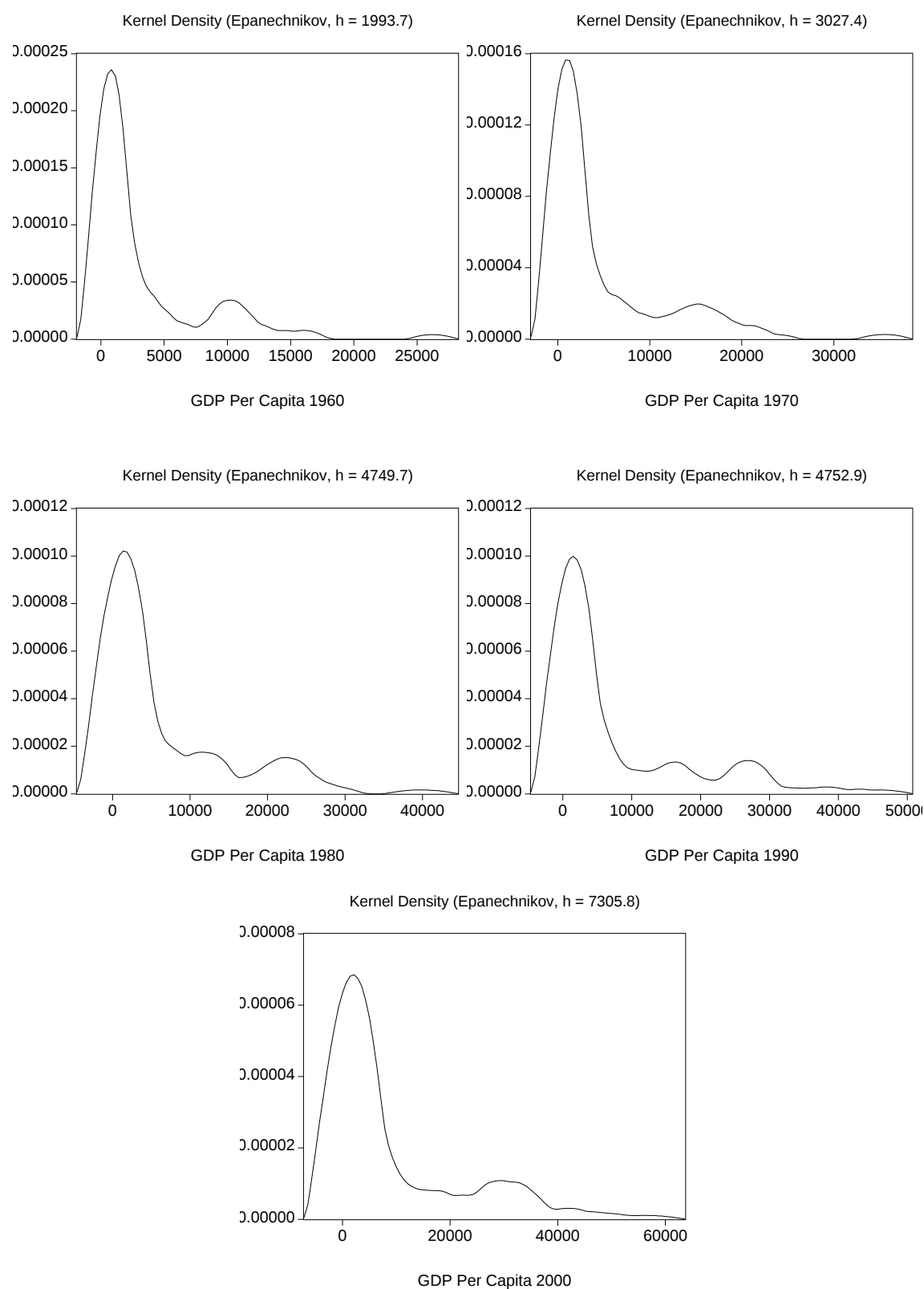


Figure A14: GDP Per Capita (constant 1995 US dollars) – *unrestricted dataset*

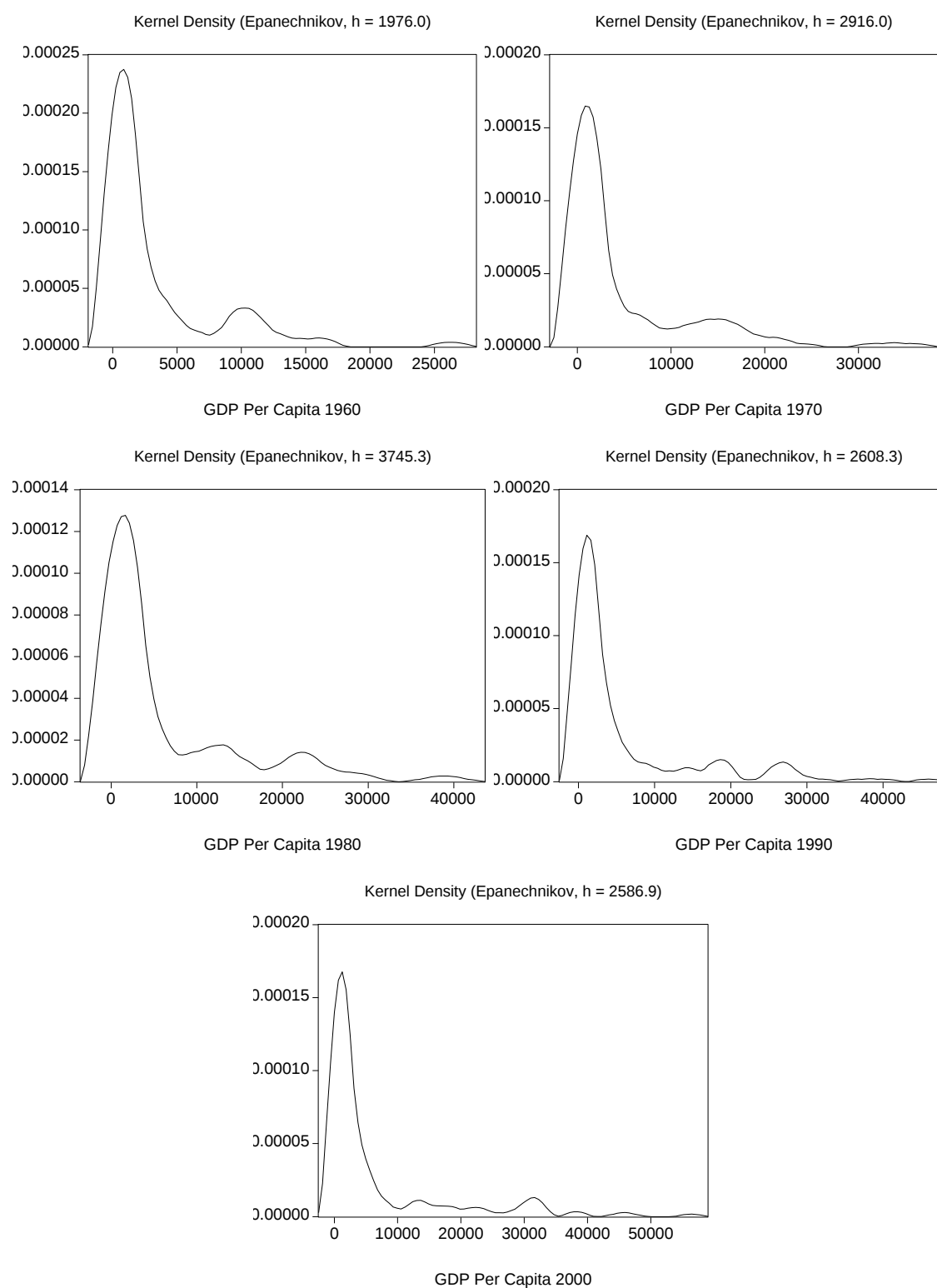
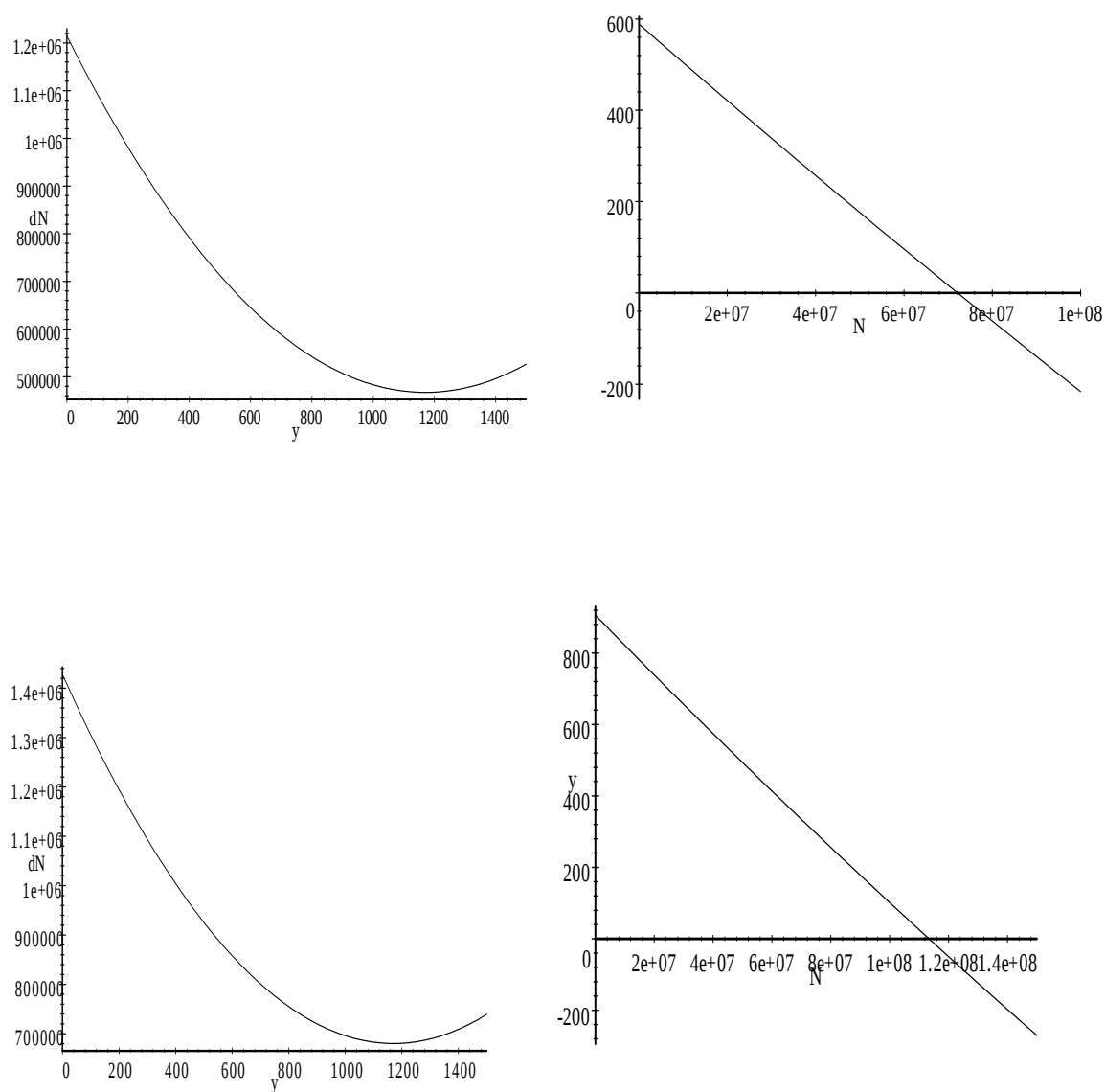
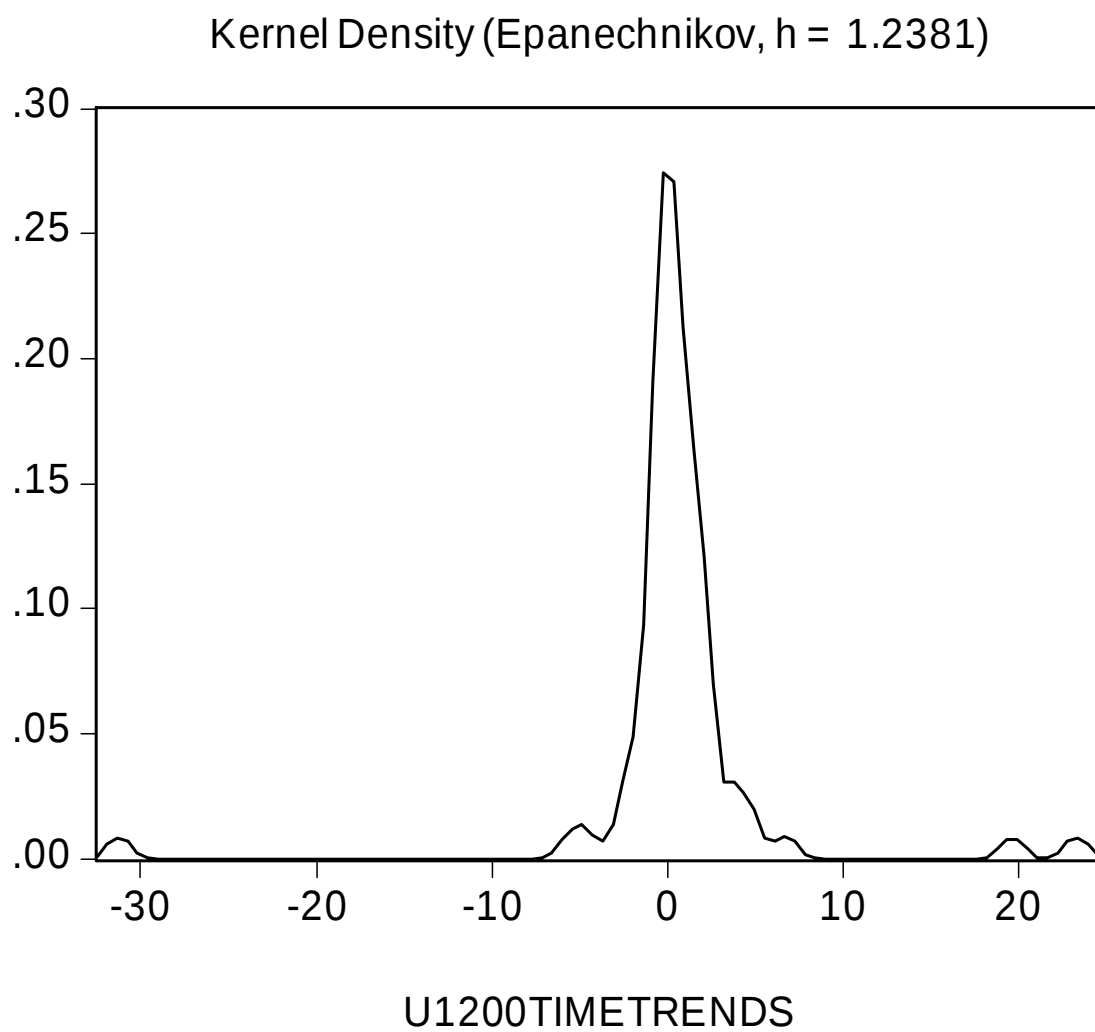


Figure A15: The U1200 group for 1960 and 2000



The plot of the estimate of Foley's model for countries with GDP per capita below \$1200 for the years 1960 and 2000 exhibits Smithian population behaviour and decreasing returns to scale. Both curves shift upward over time. The panel average values are $y = \$475$, $N = 38\text{mln.}$, and, for the years 2000-2002, $y = \$494$, $N = 52.5\text{mln.}$

Figure A16: Distribution of coefficients of time shifts in the production function of the u1200 countries



Appendix B: Forecast Figures (NOT FOR PUBLICATION)

Figure B1 World Population forecast

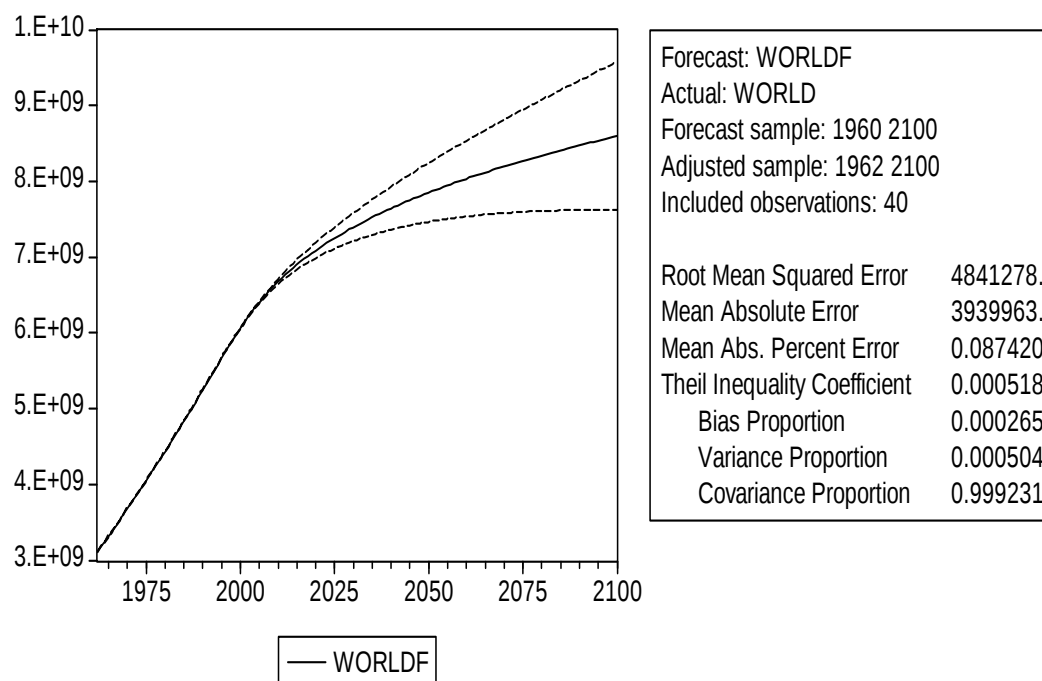


Figure B2 Population forecast high income countries

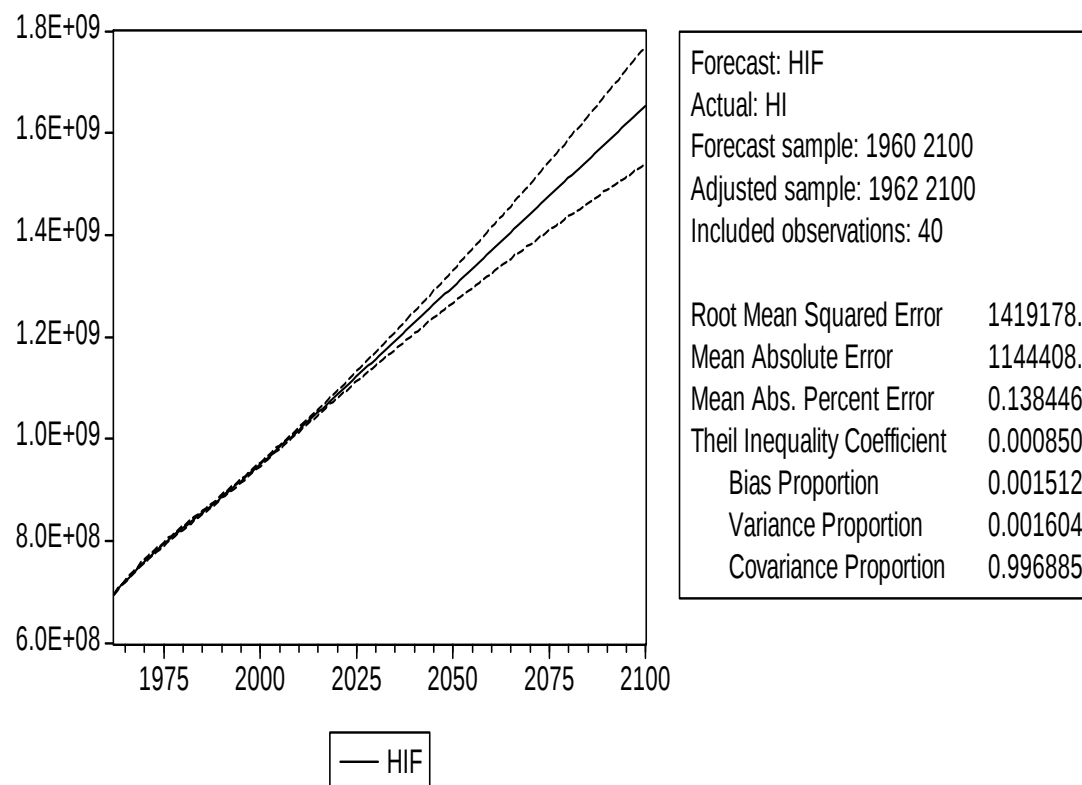


Figure B3 Population forecast Upper Middle Income Countries

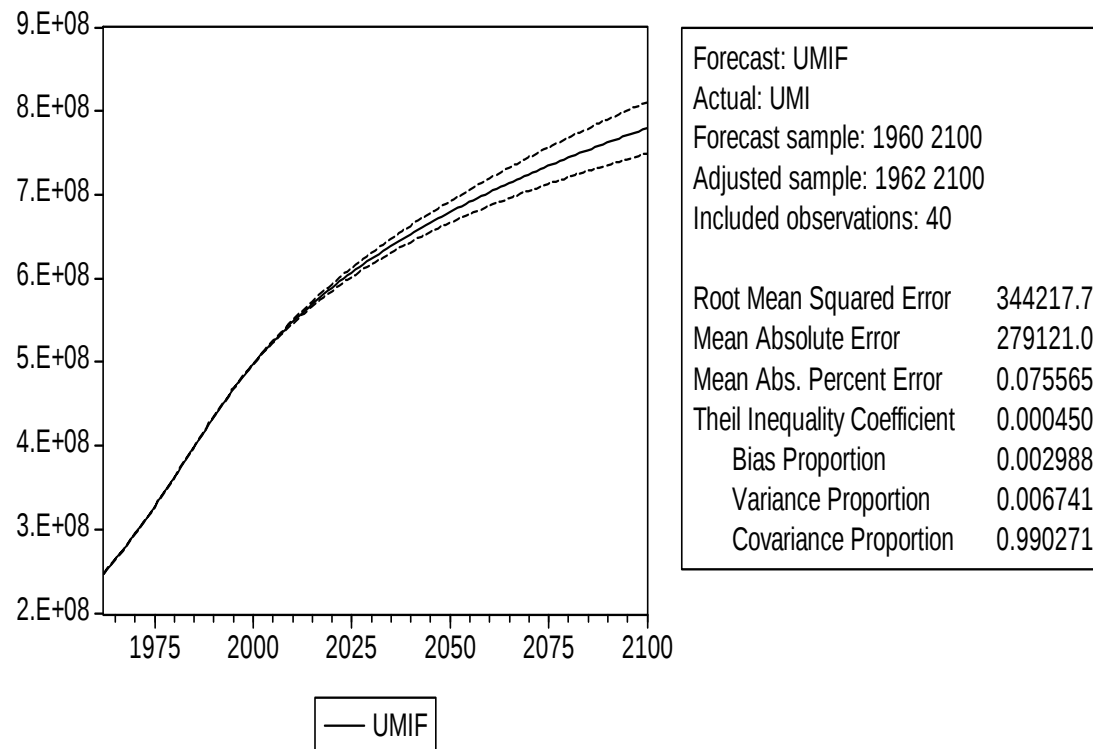


Figure B4 Population Forecast Lower Middle Income Countries

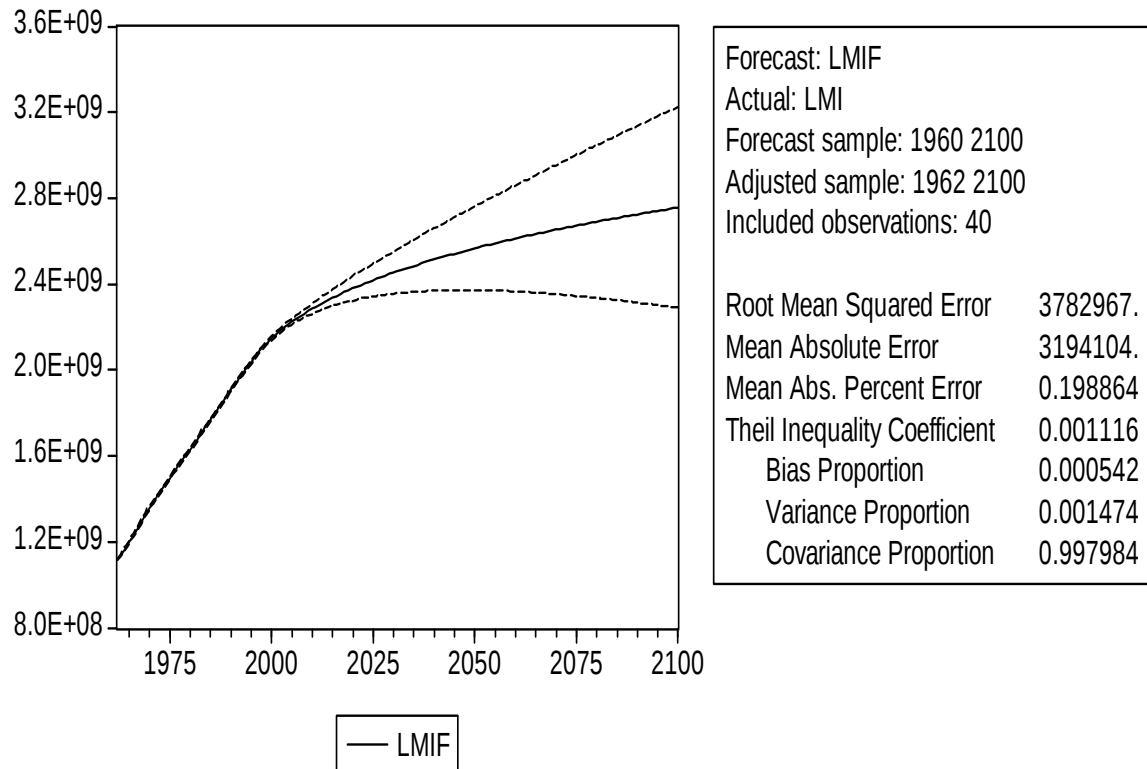


Figure B5 Population Forecast Lower Income Countries, (1)

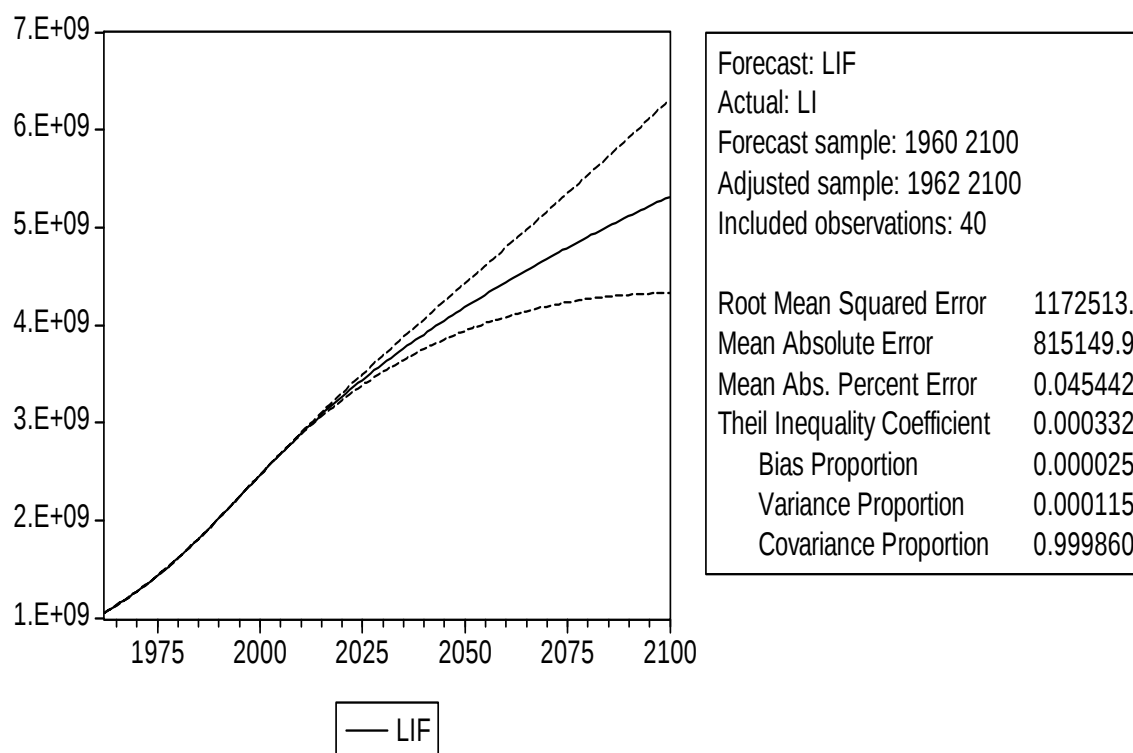


Figure B6 Population Forecast Lower Income Countries, (2)

